Imre Toth
"Deus fons veritatis": the Subject and its Freedom. The Ontic Foundation of Mathematical Truth

A biographical-theoretical interview with
Gaspare Polizzi

Abstract: *"Deus fons veritatis": the Subject and its Freedom. The Ontic Foundation of Mathematical Truth is the title of Gaspare Polizzi's long biographical-theoretical interview with Imre Toth. The interview is divided into eight parts. The first part describes the historical and cultural context in which Toth was formed. A Jew by birth, during the Second World War Toth became a communist and a partisan, enduring prison, torture, and internment in a concentration camp from 1940 until 6 June 1944. In the second part Toth presents his mathematical training as a "vocation" that led him to rethink the whole tradition of mathematical thought critically, on the basis of non-Euclidean geometry. In the third part Toth describes his research in the history of mathematics, which begin with his studies on Aristotle and mathematical thought and on Plato and the negative ontology of the irrational recognizable in the theory of the infinite dyad and of the One. In the fourth part Toth criticizes the positions of Frege, who came to deny non-Euclidean geometry, viewing it as an expression of irrationality and mysticism.*
In the fifth part Toth maintains that mathesis and poiesis have similar ontological structures, and he speaks of his collages métaphysiques. In the sixth part Toth recalls how the birth of the idea of freedom made possible the highest political, social and artistic achievements, as well as the entire movement of human emancipation. And this was thanks to philosophy, which is not a science but a knowledge of the subject on the subject's part. In the seventh and eighth parts Toth speaks of the value and the role of mathematics in the affirmation of the phenomenology of freedom, and remarks on his relations with French and Italian cultures.

1. Roots, War, Persecution

Polizzi: Let us speak of your "first life" which ended during the Second World War with your spell in prisons and concentration camps. Roots are important for everyone, but for you this is perhaps particularly true. Your family, the Roths, were Hungarian Jews; your father Abraham fought in the First World War as an official in the Habsburg army; you were born in Szatmar (today Satu Mare), on the border between Romania and Hungary on 26 December 1921; your parents had sought refuge there while fleeing the 1919 *pogrom* in Hungary, with the intention to emigrate to America. What can you tell us of your family and the early years of your life in this town in Transylvania?

Toth: You want *subjective* information. It isn't pleasant for me to speak of these things, which are mainly private. I will say this openly: in all these happen-

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ings I am, so to speak, completely *innocent*. At that time it was normal for Jews to be persecuted; it was shameful to be Jewish. I belong wholly, *volens nolens,* to this group of mankind. Since then things have changed radically. Today the word "Jewish" is surrounded by an aura of the innocent victim, sacrifice, and it seems to entail respect and compassion. But I, personally, deserve this in no way. The feeling I get is that I am abusing the good will of those who privilege me with their compassion.

My father was a great believer. He fought in the cavalry during the First World War on the Italian front on the Isonzo. I have a postcard dated 1 November 1916 sent to his fiancée, my mother, who was eighteen and in the Red Cross at the time. My father is portrayed in his official's uniform as he examines a map – of the front line of course – with his tent behind him. Underneath the picture are the words: "I'm looking for the path leading to peace," and above is the army censure stamp: *K.u.k 12. reitende Artilleriedivision.* He had fought in the war, he too had some important medals which in 1944, upon a pledge by the Fascist government, should have given their holders special status. But in 1944 my father refused to request the exemption from deportation that was given to Jewish former soldiers – which, however, meant nothing as they were taken away without hesitation just like all the rest. The deportation operation in Hungary was exemplary, perfect; six hundred thousand Jews were taken away in around two weeks. My father often spoke of an episode that had shaken him; one day with his commander, who was also his friend – I can still remember his name, Tarnay –, a very self-assured painter, he had gone into a Catholic church; his friend wanted to show off so he unsheathed his sword and cut down a canvas portraying the saints. My father told him that he shouldn't have done it. When they came out, an old Italian peasant, who had seen what had happened, shouted at the official and cursed him; two days later commander Tarnay died. Of course it is easy to die in a war, for many reasons… But this episode made a big impression on my father, who was not superstitious, but a great believer, and he told it to me several times as a boy. He did not hide from me that he took the death of his commander to be a sign from on high.

Polizzi: What role did Jewish traditions, culture and religion play in your upbringing? What schools did you go to before you went to university?

Toth: I studied in a Catholic Episcopal college; this was quite normal, not out of the ordinary at all. I also read the philosophical works my father had (Kant, Nietzsche, the pre-Socratic philosophers, Schopenhauer…). Nearly every Saturday afternoon my father read passages from Spinoza's *Tractatus theologico-politicus*, something which struck me as odd. I asked him: but how can it be that you, father, such a staunch believer, can read and appreciate Spinoza and his *Tractatus* of all things? He answered me with a mysterious smile: I think this is the book that every true believer should read. We were heading towards the late 1930s – I can picture his face in front of me, as if it had happened today – I was bowled over; this really quite unexpected answer left me totally dumbstruck. However, the book he considered the supreme peak of human thought was Kant's *Critique of Pure Reason*. Whoever understood this text had reached the top – he often repeated to me, with a look of excitement and true adoration in his eye.

I got interested in maths at college. We were a good group of pupils, but I annoyed the teacher because I asked questions he thought were pointless and irrelevant. And indeed my questions were about – I now realize – the problems of the axiomatic, but also ontological, foundation of mathematics. At high school, a third of the pupils were Hungarian, they were more "cultivated," they were familiar with the national literature, famous novelists like Dumas and Verne, but reading was not one of their social values; they were good-looking, athletic, intelligent even, but they believed more in manliness, sport, being successful, rich, they spent their time with the girls, drinking, going hunting, playing football. A third of them were Romanians, they were lazier, haughty, not very sporty, less "cultivated." But both groups laughed in the face of culture and reading. They put on all the airs in their fancy get-ups! Their dazzling dances were spectacular and we, the Jews, admired them with a quiet, but very deep and sincere jealousy. Both groups derided the culture and so-called modern reading of the Jewish boys and both expressed their deep disgust and hatred for the cosmopolitan ways of these Jews without a fatherland, without roots.

Their values belonged to the provincial nobility: physical courage, pride, chivalry, respect for social hierarchy, patriotism, obedience. We Jews greatly admired these values, but at the same time we were perfectly aware that we were incapable of making them our own. We were jealous, pale, wan, weak, with a physique that was not attractive at all, the eternal losers, visibly fearful, pretentiously humble, perceptibly insolent – quite an irritating mix which definitely did not help us get accepted. Our envious admiration could by no means hide our disdain for their way of living. We were oppressed, persecuted, we were without doubt considered disgusting worms, public humiliation, which everyone felt to be motivated and legitimate, was part of daily life; yes, we were victims, but I must confess, definitely disagreeable victims who were very difficult to swallow. We were wrapped up in culture, by the novelties appearing in every field: in physics there was the relativity theory, in quantum physics the uncertainty principle, and then there was psychoanalysis, the Oedipus complex, Marxism, class struggles, imperialism, serial music. In even the poorest Jewish households, there was always a piano and someone who could play.

In 1940 a friend of mine, Eva Braun, strangely, a typically Jewish name, – a very beautiful girl with enormous doe eyes – played Schönberg's transcription (or maybe it was Alban Berg's, I can't remember) of Bach's *Musical Offering* for me. In 1943 I was in the military prison in the town of Kolozsvar (today Cluj, in Romania) where I received a letter from a friend, Judith Kosa, a girl from Budapest (she worked in a photo lab). It was a long, four-page letter, to cheer me up, with a detailed and enthusiastic analysis of Beethoven's opus 111, which she knew by heart. She did not survive the war: as I found out when I left prison, in 1944, she was brutally strangled in the streets of Budapest and her body thrown into the Danube.

We read modern writers, the Soviet novelists, the writers of Germany's Weimar Republic, Thomas Mann (*Joseph and His Brothers* was my father's constant read, in our circles we passionately discussed his recent *Warning Europe*), Stephan Zweig (my sister's favorite), Lion Feuchtwanger; we read the Americans Upton Sinclair, Jack London, John Dos Passos, whose poetry/prose we found fascinating; America's black poets such as Langston Hughes; France's André Malraux, Romain Rolland, Roger Martin du Gard; Italy's Pirandello, Marinetti, Papini, Ignazio Silone – I used to read pages of *Fontamara* aloud to groups of peasants, poor people who could not read or write, from the villages around the city; the novels and surrealist *pièces* by the excellent Czech novelist Karel Čapek – in his utopian drama *R.U.R* I was amazed to come across the word "robot" for the first time and the idea of a future with Rossum's Universal Robots; I dearly loved the great Czech poet Jiri Wolker and I read and recited his famous poem *Dans le Café Metropol* in Czech; we read the British writers Bernard Shaw, Aldous Huxley, the *Fabians*, H. G. Wells – his technocratic utopia was discussed with the same interest as Marxism; we knew the Russian Constructivists, the German Expressionists.

It was boys and girls together, with no distinction; we discussed the latest trends on the cultural scene, we accused each other of reading or not reading certain books – the *Critique of Pure Reason* or Freud's *Totem and Taboo* – or of knowing or not knowing the new artistic currents (Impressionism, *Pointillisme*, Futurism, Cubism); you can imagine our level of culture was very different from the majority of students. We had a left-wing Hungarian magazine which covered all these books and knowledge, it was the best left-wing emigrants' magazine, it cost a lot and was directed by a great academic, a truly ingenious editor, Gabor Gaal, who was Lukács' deputy when he was people's commissar during the Hungarian Commune. This magazine, *Our Age* (*Korunk*), promoted an extraordinary spirit of modernity, it was very serious and all the best left-wing intellectual emigrants from Hungary and in part Germany wrote in it. One of the great biologists of the time, Ludwig von Bertalanffy, spoke of the interaction between vitalism and mechanism in biology, other authors presented the theory of relativity, quantum theory, Modernist currents, Russian Constructivism. All the most famous writers contributed to the journal: the great Constructivist painter and sculptor, Moholy-Nagy, another poet and Cubist painter, Kassak, as well as Bela Balazs, one of the first cinema theorists. In the Eastern European countries – Hungary, Poland, Russia, Romania, the Baltic countries – being Jewish meant belonging to a certain social group. I was an atheist, a Communist, for me the Jewish religion did not mean anything; nevertheless I was organically and indissolubly part of this social group, which was also a religious group, though the religious side was not important because what counted were certain traditions, certain values; above all reading, books, knowledge and culture. In the Jewish communities knowledge was one of the supreme social values; it was this sensitivity, perhaps excessive even, towards cultural values, everything that was modern, that glaringly set the assimilated and liberal community of Jews from these countries apart from all the other communities in the population. In this period I was part of this small group of young people who took seriously books and culture, not just hunting and sport, and I was very interested in the theory of relativity. I remember that I was not keen on things connected to nature, to *empeiria*; I found the theory of relativity interesting because of its geometry, because of Minkowski's geometry, which fascinated me. This is where my interest in non-Euclidean geometry stemmed from.

At the beginning of the 1940s, the Jews were excluded from practically everything. Some young Jewish musicians organized a symphonic orchestra in Kolozsvár (the capital of Transylvania). The Jewish community lent a squalid but very large hall to the orchestra where they rehearsed Bizet's *Arlésienne*. I went to the rehearsals upon invitation of my friend György Ligeti, who was later to become one of the greatest contemporary composers. Ligeti played the drums, the percussion instruments played an important role in the score but came in very rarely. In the long intervals, between two thundering drum beats – when it was his turn George got very wound up and went all red – we passionately discussed Heisenberg's uncertainty principle, getting the impression that we had understood something. Despite being a musician by vocation, not only was Ligeti greatly interested in scientific discoveries – quantum physics, relativity, Gödel's theorem – but he was also very up on their technical details and mathematical stratagems. Besides, he was not the only one, nor by any means an exception.

Polizzi: In the 1930s anti-Semitism took a strong footing in Eastern Europe (and elsewhere), the place where there were the most Jewish communities. Racial laws against the Jews were also issued in Hungary, for example in schools there was the *numerus clausus*, which had direct and indirect consequences on your future.

Toth: In the 1920s, immediately after the First World War, the first racial laws in the history of Hungary were issued; as a result, there was a large migration of Jewish mathematicians, physicians, psychologists and scientists to America (John Von Neumann, Theodore von Kármán, Edward Teller, Leo Szilard, Eugene Wigner…); among the physicians who emigrated to the United States there were no less than six or seven Nobel prize winners of Hungarian origin. In Hungary – as would happen later in Germany – racist laws proved to be the strongest means of spiritual counter-selection: all at once these countries got rid of their best men in the fields of science and culture. My father was part of a regiment that went over to the Communists after the war. He wasn't a Communist, he was a supporter of Socialism, not Communism. But he was an official in a red regiment and after the revolution was overturned, the counter-revolution spread terror against the Jews, against the Jewish-Bolshevik peril. In the West they were very scared that this blaze of Communism would break out everywhere, after what had happened in Russia. In Hungary, the predominating opinion, upheld by most of the public, was that the Jews alone were responsible for the defeat of the Austro-Hungarian army. Our family had to emigrate and moved to Transylvania to flee persecution from the Hungarians. The first big *pogrom* against the Jews had taken place in Hungary straight after the war, even though the state – and Admiral Horthy in particular – did not promote the initiative; Horthy was very Austro-Hungarian, a great law-abider and did not want to take initiatives that were not justified by the law. The persecution was backed by the extreme right. Transylvania had been a very important Calvinist principality in the past; it was where the Hungarian literary language had originated. In the 18th century it had become a province of Austria, and then of Hungary. But after Hungary became Communist, in 1919, Transylvania was handed over by the Western powers to Romania, which was the last state to grant, in 1923 – after much hesitation, obstruction and delay – citizenship to Jews under pressure from the West. We went to Transylvania when it was already Romanian, then it went over to Hungary again during the war, and then back to Romania. We moved to Satu Mare to start a long journey; my father wanted to go to America, and as a result, every evening we would sit around the table and learn English. My father gave me English lessons, he made me read and learn English children's poems, but then the Great Depression arrived and the project went up in smoke.

Polizzi: The war upset your family life and led to highly important choices that scarred a whole generation. Jewish, Communist and partisan: a total commitment. How much of your commitment was down to chance or circumstances? Can you list the reasons that led you to make those choices?

Toth: None of my friends were Communists; they were very religious. But in Hungary there was a great Socialist tradition of literature and thinking, like in Italy or Austria. In general in Jewish circles there is a very wide-ranging and intense sensitivity toward problems of social justice. And hence there was a great interest in Socialism. This wealth of Hungarian leftwing literature was part of the basic self-education of – almost exclusively – young Jewish intellectuals. And for me it seemed very normal to choose Socialism, and later Communism. Of course, in that period of the rise of Nazism, mine was not a very original choice among young Jewish intellectuals. It's strange: as a boy, it seemed the most natural thing in the world. There's another reason too. I felt guilty in the face of poverty and servitude. One day, when I was a child, I got on a train with my mother and I saw the miserable, half-naked railway workers putting the rails under the train by hand; I felt guilty because I was on the train that was running on the rails that these workers had laid. These workers who came to work barefoot, with their shoes round their neck so they wouldn't wreck the ballast, who worked for the humblest of wages. Today I would say that I lived in the relative poverty of the petit bourgeoisie: there were four of us, plus an old aunt, living in one room with a kitchen attached. There was no running water, but then I was strongly convinced that we were rich, that we were at the top of the ladder. And indeed we did have the privilege of electricity, while the others had gas lamps and there were big families who lived in a single room without a separate kitchen. Hence, I had the strong feeling that I was privileged, and that I didn't deserve it. A very powerful sense of guilt, and revolt. Today it seems odd, very odd: a twelve-year-old boy living under the heavy burden of guilt due to the poverty of others. Nevertheless, it seems odder still that then I considered my guilt a totally normal and natural reaction and that I was convinced that everyone felt as I did; I felt real compassion for the wealthy who must have felt – I was convinced – truly unbearable guilt. Isn't it ridiculous? I couldn't bring myself to believe in the Jewish religious rites and I was very scientific, materialistic and Communist. I began very young, I was an early starter: at the age of 13 I became a Communist and at 15 I was already a militant. But don't use the word "courage"; I was rather overcome by a feeling of fear, the permanent underlying feeling that has accompanied me continuously throughout my life. Despite my choice seeming the most natural thing in the world, a real *categorical imperative*, to use a term that is definitely a bit much. And I have to say that I was aware of the trials against the Trotskyites and that I did not have the slightest doubt in my mind as to how they were organized; true, what I knew upset me deeply, and a good dozen years later it played a determining role in my abandoning Communism.

When these farcical trials were organized after the war in Hungary, Czechoslovakia and Bulgaria (in Romania a similar farcical trial took place after Stalin's death), I realized that it was impossible to form a Communist power without this type of state terrorism. But then I made myself close my eyes; this information could not hinder my determination, I didn't even consider making a different choice. Rising up before me was the ever more powerful, ever more aggressive, blond *triumphant beast*, its hyena's cackle making Europe's walls tremble, its animalistic violence, its blind bestiality becoming more and more obvious, its pestilence filling the air: it spewed its coarse words right and left, announcing the end of Western culture. I was utterly exasperated by the lack of a real reaction. All the bourgeois or religious political parties gave in, they all attempted to tame the *Führer*, to keep in his good books. Aged sixteen at the time, I was totally convinced that the Communist movement represented the only incorruptible force that could effectively oppose Fascism, racism and anti-Semitism. It was no time to hesitate; I heard a deafening voice shouting out in my ear: *No*! *The shame of it*! If I am now to cast a glance at my past, I realize I was different, perhaps even absurdly "odd," but then it was normal for me – no, not just normal, much more than that: it seemed irresistible, and utterly necessary,– to act in that way. It seemed totally unthinkable that one day "Real Socialism" could turn into poverty, oppression, crime, and, naturally, anti-Semitism.

Polizzi: Prison, torture and internment in a concentration camp from 1940 to 6 June 1944, hospital and the incredible conclusion to your imprisonment. A dramatic time that pushed you to the very brink of survival.

Toth: Ours was a small, paltry concentration camp. And we must not forget that the camp was not German but Hungarian, and that makes a big difference. There were only a few of us and we didn't live like in Auschwitz; but you could die there too. In these small Hungarian concentration camps, you could paint, play music and sing, you could do various activities. I absolutely devoured books. While I was in the camp and afterwards in prison, I filled more than thirty notebooks, many of which I still have; I started the first on 7 November 1941, on the anniversary of the Russian revolution, with a beautifully drawn quadrature of Archimedes' parabola. Graphically it was very beautiful: geometrical constructions in three colors of India ink: red, green and black. And all this in a concentration camp; incredible, inconceivable, isn't it? During an execution one of my fellow prisoners said to me: "You're a monster, while they're killing one of us, you're doing geometric drawings, you're monstrous!" Then I was taken aback by these accusations. Only now have I realized that my "love for geometry" was as perverse as that of Don Juan, in Max Frisch's novel *Don Juan, or, the Love of Geometry.* But then, in that dull little hell that was our camp, I didn't notice; without worrying about it, I sank into the blissful unawareness of my heavenly innocence. I was twenty. I spent my time in the concentration camp reading and studying; my attitude was strange, cynical even, perhaps… But today I realize that it saved my soul, it saved the inner me…

Polizzi: Primo Levi says that too in *If This is a Man*, when he tells of the effort to remember the Canto of Ulysses in Dante's *Inferno* as an attempt to rise above the desolation of imprisonment[1].

Toth: Primo Levi said that too? Afterwards I realized how true what my friend said was: it was monstrous, yes, even though our camp was not Auschwitz. But, of course, it was no easy ride! It was easy to be killed there too, killing wasn't a problem, even though there was no hi-tech industrial mechanism like at Auschwitz. One fine day, one of the most outstanding figures in the Hungarian Communist Party, Mihaly Hay, an unforgettable friend who was my father's age, died like that.

Polizzi: How much did these four years affect you intellectually and politically?

Toth: It was an experience, a school, an education, for everyone, without doubt. But for me, personally? I believe it made me more mature, more tolerant – before I was intellectually intolerant. Perhaps it also made me grow up.

Polizzi: You tell of how when you returned to your ravaged home after being freed from the concentration camp you found your books, amongst which many on philosophy, untouched thanks to a beautiful letter that your father left to the Nazi raiders asking them to save them, remembering the episode of the death of Archimedes. You were saved thanks to mathematics; you began your "second life" by studying mathematics.[2]

Toth: That's quite right, though I wouldn't call it a "second life."

Polizzi: I'd like to go back to your being Jewish again, something that is by no means limited to the catastrophe of persecution and massacre by the Nazis. Not only is it a difficult wound to heal in your personal and private life, it

[1] See P. Levi*, Se questo è un uomo*, Turin: Einaudi, 1992, pp. 133-141; [*If This is a Man*, trans. S. Woolf, New York: The Orion Press, 1959, pp. 127-134].

[2] "My second life is perhaps that of a maniac, because you have to be a maniac to do all these things in prison, in a concentration camp." See R. Gatto (ed.), "Colloquio con Imre Toth," *Lettera Matematica Pristem*, 6 (1992), p. 9.

is also a deep wound in Western society and culture, whose epoch-making importance has been underlined by the most informed intellectuals[3]. In May 1997 you gave a paper at the convention held in Naples on *The Shoah between Interpretation and Memory*; and I repeat the question that you chose as the title for your paper: *What does it mean to be Jewish after the Shoah*?[4]

Toth: You're right. A deep wound in the Western spirit, but also a self-inflicted wound caused in all awareness. Anti-Semitic hate is as deeply rooted in Western traditions as democracy, human freedom, justice and loving your neighbor. As, I might add, the Jew himself. Indeed, Jews are the only fossil to have survived since the Roman Empire. Before the war it was shameful to be Jewish, it was difficult to accept and it had to be kept hidden. Many Jewish friends of mine converted to Christianity. But if they thought they'd get out of the concentration camps, they were wrong: it counted nothing whether you had converted or not. I remember the case of the musician Schönberg. He had become a Christian and in the 1930s he had wanted to convert back to show his sympathy for other Jews. But the rabbi in Paris told him that for the Jewish faith there was no need to convert back because he had converted under social duress and as a result, deep down, he had never stopped being a Jew. The two witnesses to this meeting were Picasso and Chagall. In 1941 Bergson was another who did not want to convert to Catholicism to remain close to the persecuted Jews in Paris. But many people did convert: Edith Stein or the famous French poet Max Jacob. The Nazis made it a matter of honor to find them, even in the monasteries, as happened to Stein and Jacob. It didn't count, but then lots of people tried to hide their faith.

Polizzi: Still sticking to this topic, which is so essential in order to understand the culture and philosophy of the second half of the 20th century (a list of just some of the great Jewish thinkers that lived through the *Shoah* and wrote about their experiences contains such names as Adorno, Horkheimer,

[3] See E. Traverso, *Le radici della violenza nazista. Una genealogia*, Bologna: il Mulino, 2002.

[4] The conference was the international convention *Olocausto. La Shoah tra interpretazione e memoria* [The Holocaust. The Shoah between Interpretation and Memory], Naples, 5-9 May 1997, organized by the Istituto Italiano per gli Studi Filosofici in partnership with the Philosophy Departments of the University of Naples Federico II, the Milan State University and the Collège des Etudes Juives; the *Atti* were published by Paolo Amodio, Romeo De Maio and Giuseppe Lissa under the title of *La Shoah tra interpretazione e memoria*, Naples: Vivarium, 1999. Toth's paper is now in print under the title of *Essere ebreo dopo l'olocausto,* Fiesole: Cadmo, 2002, as part of the series *L'orizzonte della filosofia* directed by Romano Romani, whose kind hospitality on a beautiful day in Rome first gave rise to this interview (29 September 2002).

Marcuse, Lévinas, Arendt, Weil…), I would ask to what extent you took this experience into account in your choice of studies and direction of thought. The political dimension of the being of the human condition can also be found in "abstract" fields of thought such as mathematics.

Toth: It is said that Jews never managed to integrate, were never accepted and that in the end they were killed off. I think that we experienced an immense defeat, but that we acted with great strength in trying to integrate and that, despite not achieving total integration, this plan and the effort to do so nonetheless produced some results. Caesar, Charlemagne, or the Catholic Church had a big political plan, but they never totally accomplished it – and the same goes for the Americans with their "American dream" – but even though they did not fully achieve it, we can't say that they didn't do something that is still an integral part of the West's spiritual treasure. So, the Jews wanted integration, but they didn't manage to fully accomplish it; the Germans, the Hungarians, and the Russians never accepted this. But let's think of the contribution that Jews have given all the same to all branches of 20th-century culture: medicine, physics, art, philosophy, literature, psychology, sociology, poetry… I don't want to reel off the names, but I can mention Einstein, Schönberg, von Neumann. How many women of Jewish origin have played a predominant role in science, art and philosophy in the 20th century! Just think of Hélène Metzger, Hannah Arendt, Simone Weil, Edith Stein, Else Lasker-Schüler, Emmy Noether, Lise Meitner, Anneliese Meier, Rosa Luxemburg… There is no doubt that the mass participation of women in intellectual life in the last century seems to belong almost exclusively to Jewish circles. It is certain that in proportion to a population of around 18 million people (remaining after the massacre of around 13 million), perhaps no human group has ever contributed to culture to such a degree. But, it is said, they killed more Jews in the last century than in the whole of human history; what counts for us are the dead, the millions of lives that we lost. And this is what I say to that: "Yes, you didn't manage to integrate, but that was impossible; all the same you have made perhaps the greatest contribution to human culture in the whole of the 20th century." I'm told that I'm cynical, that we don't need to stake these claims, because at the same time there was the massacre. They don't accept my "cynicism," but, like it or not, history is cynical. I read the documents allowing Einstein to study at the University of Berlin in 1913. They read, "although he is a Jew, we think we can accept him…," signed by Max Planck. So, the Germans did not accept the Jews? The Jews became an integral part of German culture, of Western thought, all the same. Perhaps also the first truly "European" human group in the present-day meaning of the word.

2. Education and Early Studies. Mathematics and Its History

Polizzi: You started your academic life by studying mathematics, and immediately an interest in non-Euclidean geometry came to the fore. Could you retrace how this dedication arose, also in relation to your knowledge of relativistic physics?

Toth: First of all, I'd like to say that I'm no mathematician. I studied maths at school and I was always one of the three best pupils. I always figured among the groups of the best students, but the best mathematics students really become mathematicians, researchers. I liked studying maths, new, modern theories, but what interested me in particular was the speculative side, the new, spiritual factors that mathematical concepts, at times so bizarre, brought about. I started to ask myself questions that struck me but my teachers did not understand. My mathematical colleagues ask me: "How could you have asked yourself these questions?" I asked my teacher, for example: "Yesterday you told me that it is impossible for a number multiplied by itself to be minus one, because in that case the number would neither be positive nor negative nor zero; today you're telling me that by multiplying two imaginary numbers we get a negative number. How can that be?" Indeed, how can the impossible be possible? The teachers didn't answer me; they said I was cheeky and that mathematics did not answer such stupid, childish questions. Back then I hadn't discovered non-Euclidean geometry, I only found out about it during the last year of secondary school. The discovery of non-Euclidean geometry was linked to the theory of relativity.

Polizzi: For you mathematics was not just an academic and professional choice, but a "vocation" that made you critically rethink the whole tradition of mathematical thought in historical and philosophical terms. The Bolyai University in Cluj was the first place given over to your mathematical studies, as luck would have it, dedicated to one of the promoters of non-Euclidean geometry.

Toth: At university I started to read the great mathematical classics, because I wasn't satisfied by the answers to my "naïve" questions. In Cluj (Kolozsvár, Klausenburg, *Napoca* to the Romans, and *Claudiopolis* in the Middle Ages) I studied maths. I didn't want to study philosophy because I had already read Kant, Hegel, Spinoza…, I didn't feel the need; instead I was interested in the philosophical issues of mathematics. Then the first thing I needed to do was learn the trade; I am no mathematician, but I studied maths. My professors could not motivate my interests: I was an intelligent boy, but not intelligent enough to understand how negating a negation could explain that a negative number multiplied by a negative number produces a positive one or that a debt multiplied by a debt can lead to a profit. I couldn't find anyone who could give me an explanation, because they were not interested in this speculative side of maths, just its practical side. And so I started to read the classics, the works of the great founders. I discovered the history of mathematics in a very natural way, when I asked myself these questions on the oddness of negative and imaginary numbers which made me doubt the idea that mathematics was a science based on logic. Indeed, bit by bit I convinced myself that all progress in mathematical thought was made in direct and manifest contradiction to the formal laws of inferential logic. Or, as my friend Romano Romani said to me recently, the *Logos*, which directs the movement of mathematical thought, *will not let itself be reduced to logic.*

Polizzi: Mathematics and Socialism combined in your professional and political activities after the war. From 1949 to 1969 you taught the Philosophy and History of Mathematics at the University of Bucharest, while at the same time you were a member of the Romanian Communist Party, until 1958 when you were expelled. Can you describe these two parallel experiences to us against the historical background full of great hope, often of rebirth, and the rediscovery of the freedom of research in the sphere of mathematics?

Toth: I started teaching the Philosophy and History of Mathematics at the University of Bucharest in 1949; I was a temporary lecturer. At that point I had already been accused by the party of being a Titoist, and I was already being kept in check. When I protested against injustice and the abuse of power, my friends told me I had to moderate my views, that the party was led by a superior proletarian set of morals which made the interests of the proletariat class the supreme moral value. I belonged to the same "biotype" as many others who after the war had concentrated on climbing the social ladder to power, however. I had been in the Resistance and was formally a "hero," so I was given quite a lot of respect. But little by little I realized that the ethical ideals of Socialism were all show, propaganda, that they were used to hide the meanest of interests in power and personal achievement. And I also saw something that deeply upset me, that for a long time I tried to deny. Unfinished sentences, suggestions that not only didn't sound right, but above all rang very suspicious in an arena that called itself Socialist. Nazism had built the "myth" of the Jewish-Communist plot to justify the killing, and it was said that the capitalist countries wanted to take the very same myth up again against the Communist world. At that time it was better not to give them the chance to level criticism, it was preferable not to talk of the persecution of the Jews and not to highlight the presence of Jews in the Socialist movement too much. It was said that the

Jews had to integrate, they had to become part of the people, and at the same time they were driven away, put aside. This is how part of the population, and even some Communist intellectual Jews saw it: that we were not to give the Capitalist enemy firing power, we had to hide Judaism, and not talk of these things too much. Little by little I realized that this too was in some way anti-Semitism, however underhand and hidden it might have been. It is incredible how for many years even Jews refused to admit to these anti-Semitic tendencies and accepted these common arguments.

Polizzi: Post-war Romania seemed to take a separate route towards Socialism, progressively standing aside from the Soviet direction taken in Eastern Europe. Having obtained northern Transylvania under the February 1947 peace treaty and abolished the monarchy in December of the same year, the People's Republic of Romania was at first allied with the USSR but then gradually distanced itself until it developed its own foreign policy (diplomatic relations with West Germany and Israel, dissociation from the 1968 repression in Prague) under Ceausescu (from 1965). But you had already been expelled from the Romanian Communist Party in 1958; there is a relationship, biographical even, between your choice and the Soviet repression of the Hungarian revolt of 1956, since an article of yours from 1957 stirred the reaction of the Romanian Communist Party[5]. More in general, what did Socialist Utopia mean for you and what traces have stayed with you from your experience in politics?

Toth: It took at least ten years for me to divorce myself from my experience in politics; it was very difficult, like splitting up after a great love affair. When your woman gets back a bit late and gives you an acceptable, totally valid excuse, but she's talking a bit too quickly, you don't want to believe she's cheating on you, and you always manage to find a good explanation so you don't have to accept the truth, until, finally, you realize she hasn't being straight with you. It was very difficult to live in Romania in those conditions. I have never had any real roots. I managed to change my social *milieu* from Calvinist Hungary to Orthodox Romania; for me personally it wasn't a problem. I felt the hatred between the Hungarians and the Romanians and the accusations they railed against each other; I listened to their stories and to me it sounded like the same story in two different languages. I realize that many of my friends set store by their roots, they want to find their roots, to integrate, but I have never had this kind of problem, I have never felt like a foreigner, I have never laid claim to a homeland. But the situation in Romania in the 1950s was difficult and, above all, it was not just a Jewish problem: the Romanians and Hungarians have not

[5] The article was "The Ethics of Scientific Research" (in Hungarian), *Korunk* (1957), pp. 1141-1148.

had it any easier by any means. Of course, we cannot ignore the fact that the Jews were not able to integrate there, but I have never had trouble fitting in. I have always felt like a citizen of the city of *Cosmopolis* where the official language is your mother tongue.

Polizzi: Your first writings, dating from 1953-54, seem to recall the protective angel of your university studies. They are articles that discuss the works and thinking of Bolyai within the wider framework of non-Euclidean geometry[6]. A line of research – on non-Euclidean geometry – that you have never abandoned.

Toth: I started to write on non-Euclidean geometry and its philosophical problems after the war; it was the start of my research. Looking back, we can already see all the problems that I would go on to explain later in those works, but my language was still struggling to get out; it was just the beginning, it was still not clear to me what I wanted, but my level of interest was great. Lots of my friends said: "What are you doing, you're a mathematician and you're wasting time on this useless research? Why don't you get into IT, into new technology? You could be successful, make money." Instead, I started studying history again, and began looking at what past mathematicians had to say about geometry.

3. The Hidden Treasures of Greek Mathematics

Polizzi: Your knowledge of the antiquities was functional to your research into the history of mathematics. It ended up combining with your research as a mathematician and historian, consequently giving rise to a completely new view of (not only mathematical) Greek thought. At a first, rough glance, it could be said that you were struck by what Alexandre Koyré called "the precursor virus," and in 1966 you set off to discover the hidden remains of non-Euclidean geometry in Greek thought, starting from Aristotle.

[6] See in particular "The Philosophical Implications of Bolyai's Geometry" (in Hungarian), in the anthology *Bolyai János élete és müve* [John Bolyai, Life and Works], Bucharest: Allami Tudományos Könyvkiandó, 1953, pp. 157-340, and *Johann Bolyai, Leben und Werk des großen Mathematikers*, Bucharest: Editura Techina, 1954. A study on Bolyai also appears among the writings published recently: "Von Wien bis Temesvár: Johann Bolyais Weg zur nichteuklidischen Revolution," in M. Benedikt (ed.), *Verdrängter Humanismus – verzögerte Aufklärung*, Wien: Turia und Kant, 1992. A full list of Toth's publications can be found in the "Bibliografia sistematica e ragionata di Imre Toth," in the appendix to I. Toth, *Lo schiavo di Menone. Il lato del quadrato doppio, la sua misura non-misurabile, la sua ragione irrazionale. Commentario a Platone, "Menone" 82b-86c*, Milan: Vita e Pensiero, 1998, pp. XXIII-XXXVIII.

Toth: I would start by saying that "priority," "precursor," "anticipation" are not historical categories; no one can reach beyond the horizon of the reality of their own times. Instead of smoothly flowing and accumulating ideas, the path that mathematical thought has taken seems to me to be a chain of radically, unexpectedly, unforeseeably discontinuous links, irreducible in their specificity to everything that came before. Therefore, there is no anticipation, and no precursors. The connection between these links is produced by the operation of fracture par excellence: negation. Discontinuities are abyssal fractures in the ontic domain: the instantaneous transition from not-being to being. So my research went backwards into the history of mathematics, and yet there is no relationship between my research into the concept of numbers and non-Euclidean geometry. I started to read the classics. Earlier, I had read Renaissance and modern mathematicians (Cardano, Cavalieri, Leibniz…). Reading Leibniz is like delving into black magic; while Leibniz says that imaginary numbers are "monstrous amphibians between being and not-being" but, though stunned, he recognizes their usefulness. Kepler is fantastic, a great academic who writes about mathematics in an esoteric manner like a mystic alchemist or cabbalist. The great founders of modern maths felt that these imaginary numbers were something totally bizarre, and indeed utterly impossible. I read Lazare Carnot, the great revolutionary, the great mathematician, the father of Sadi; in his *Réflexions sur la métaphysique du calcul infinitésimal* he writes that imaginary numbers are "inintelligibles par leur essence et d'une évidente absurdité, un labyrinthe de paradoxes tous plus bizarres les uns que les autres, ces êtres de raison ne disposent d'aucune raison d'être. Y renoncer – est une nécessité"; it is incompatible with science to speak of "imaginary" numbers, of these "abominable not-beings." I also read *The Analyst* by Berkeley, a brilliant, very intelligent, very sharp writer, who shows a surprising knowledge of mathematics. Berkeley says: should we men of religion speak of mysteries, we are reprimanded, but mathematics is more mysterious than the Holy Trinity. Theologians who speak of the resurrection of the soul are criticized, but this mathematics is incomprehensible magic; what is the sacrament of the Eucharist compared to the absurdities of maths, what do these mathematicians want from we theologians with their logic? We have the mysteries of dogmas, and them? We cannot raise any objections: Berkeley is not ignorant, he knows maths well. For example, listen to what he writes: "Whether the shifting of a hypothesis, or (as we may call it), the *fallacia suppositionis* be not a sophism, that far and wide affects the modern reasonings [of mathematicians] […] in the abstruse and fine geometry"; and then "how can they hang together so well since there are in them (I mean the mathematiques) so many *contradictoriae argutiae*." He also refers to mathematics as a "mystery" and the minds of mathematicians – and this is perhaps the best quotation – as being "far overgrown with madness." What mathematicians do is worse than what theologians do; Berkeley is right, no one has ever managed to confute him. They were books I needed to read to find the answers to what my professors called nonsense.

From there I went on to systematically study the Greeks. Greek mathematics was always present in my mind, it had always fascinated me; the splendid structural sophistication of Zeno's arguments, so simple and intelligible, so repugnant, yet of such extraordinary soundness and depth that he enchanted many philosophers.

Polizzi: But perhaps there is a more theoretical reason for your research into recurrences; perhaps what you want to do is set out a hidden agenda of mathematical ideas that are possible in operational terms, but submerged by a mathematical canon that has destroyed or nevertheless greatly compressed the creativity of scientists. As you yourself remarked, recalling a beautiful metaphor by Marx, "the hermeneutics of modern axiomatics is the key to decoding its beginnings."[7] In other words, looking at the past of mathematics takes on a different depth in light of today's mathematical theories (and this is the case for non-Euclidean geometry in particular).

Toth: One reason for my split from the community of science historians is the fact that my conception of history is very different from theirs. My *exergue* lies in Marx's phrase stating that human anatomy is the key to understanding the anatomy of the ape. If you don't know what non-Euclidean geometry is today, you can't understand the non-Euclidean factors in past works. This is my conception of history: if you don't know the current or 19th-century situation in mathematical analysis, you cannot understand Archimedes or Leibniz. If you aren't at least a bit familiar with modern mathematics, you cannot decode the mathematical passages in the writings of Plato or the non-Euclidean passages in Aristotle.

Instead of precursor, to me it seems more appropriate to speak of the phenomenological state of an *unhappy consciousness*: the knowledge of imaginary numbers, of non-Euclidean geometry, is already present in thought albeit as knowledge of not-being, its presence is rejected by consciousness but that same consciousness is also certain that it is impossible to get rid of it. This unusual state of the unhappy consciousness has gone on for a particularly long time in

7 I. Toth, *Aristotele e i fondamenti assiomatici della geometria. Prolegomeni alla comprensione dei frammenti non-euclidei nel "Corpus Aristotelicum"*, introduction by G. Reale, It. trans. by E. Cattanei, Milan: Vita e Pensiero, 1997, p. 412; in his answer Toth uses a phrase from Marx's *Grundrisse*: "Human anatomy contains a key to the anatomy of the ape."

the case of non-Euclidean geometry. Its content has been developed little by little, starting from Plato's Academy; some important passages, around eighteen, can also be found in Aristotle's *corpus*. In the 18th century Father Saccheri and Johann Heinrich Lambert and at the beginning of the 19th century Franz Adolph Taurinus had already drawn up texts on non-Euclidean geometry practically identical to those of Lobacevskij and Bolyai, but they gave the very same text the logical value of "false," absurd even, and the world that it describes the ontic value of "not being," impossible. But all their attempts to show its absurdity were in vain: *argumenta ab amore et invidia ducta* – this was Lambert's conclusion. The phenomenological state of the unhappy consciousness also appeared in the intimate spheres of their lives: they were possessed by a feeling of a failure in life, they realized that, despite their profound conviction that it was absurd, it was impossible to rid themselves of it.

The term "non-Euclidean geometry," introduced by Gauss in a letter to Taurinus in 1824, sets out the same text opposing that of Euclid, now given – in place of "false" – the logical value of "true" while nevertheless invariably maintaining the value of "true" already assigned for an eternity to Euclid's text. An ontological fracture: the immediate transition from not-being to being. In the first words of his epoch-making work of 1868, *Interpretazione della geometria non euclidea*, Eugenio Beltrami underlined that the new non-Euclidean truth had not undermined but – with his very own work – had rather confirmed and strengthened the Euclidean truth, which had been established for an eternity. The act of founding non-Euclidean geometry therefore follows the path of an *Aufhebung* in the Hegelian meaning of the word: *negating, putting an end to, preserving.* The axiom of Euclid's parallels, proposition E, is formally *negated* but its initial truth is preserved and, by giving it the same value as "truth," its negation, non-E, is elevated to the dignified rank of an axiom of non-Euclidean geometry[8]. The founding of non-Euclidean geometry meant invalidating the logical axiom of non-contradiction. The axiom maintains its validity within each of the universes, the universe can exist if and only if it is coherent from a logical point of view. The great new idea introduced by non-Euclidean geometry is the invalidation, in the *intermundia* of thought, of the logical axiom of non-contradiction: the value of "true" is simultaneously given to the Euclidean proposition E and its formal contradiction, the non-Euclidean non-E proposition; "being," the ontic value of actual existence, concerns the two opposing

[8] Euclid's postulate E states that *all* pairs of straight lines on the same plane in oblique positions to each other meet if they are stretched to infinity, no pair of oblique straight lines on the same place is parallel; its formal negation, non-E, Lobacevskij's non-Euclidean axiom, states the opposite; *not all* pairs of oblique straight lines meet, there are some pairs on the same plane that are *parallel straight lines*.

universes simultaneously. Each of the two universes also contains all existing geometrical objects: all straight lines, all triangles, all squares, without exception, are present in the two worlds. And what is more, the absolute meaning of the term "straight line" remains unchanged in the two opposing geometries: the non-Euclidean straight line is equally as "straight" – and never curved – as the Euclidean straight line. This idea of the concurrence of the contradicting truths, E and non-E, does not have any precursors, it is not the result of a slow and continuous evolution, its appearance marks an abyssal cleft along the journey of thought. It was a revolutionary upheaval, in the most proper and profound meaning of the word, a revolution that goes beyond the limits of mathematical thought – a revolution in the celestial spheres of the universe of thought: the idea of the concurrence of contradictory truths.

Polizzi: I would like to go back to that historiographic and epistemic movement which seems extremely significant in forming a history of science free from any leftover traces of positivism and progressive continuity. In the history of geometry we can make out an astutely compressed "negative" line which was only to consciously emerge in the 19th century. The result was a leap forward in the quality of geometrical knowledge thanks to mathematical freedom becoming established as the dialectic threshold for mathematical truth (to recall Cantor's celebrated remark, "the *essence of mathematics* resides in its *freedom*").[9]

Toth: I had this aphorism by Cantor in mind; as a matter of fact, it is a reformulation of Hegel's famous statement that "the essence of Spirit is freedom." I soon understood that it was not only an aphorism to recite in jest, but that it concealed a profound truth of mathematical thought. The history of mathematical thought is diffused through a negative epistemology and ontology. Of all the sciences, mathematics alone openly shows this singular feature of the subject, that is, negativity. Negation is indeed one – one of the most remarkable – manifestations of the subject, of the subject's freedom.

Polizzi: There has been a great deal of debate over your studies on geometry in Aristotle, which attest to the presence of non-Euclidean arguments in various parts of Aristotle's works (in his *Prior and Posterior Analytics*, and his *Ethics*). These studies, made public in an article from 1966, are contained in what is considered your greatest work – *Das Parallelenproblem im Corpus Aristotelicum* (1967) – and they were presented in Italian in *Aristotele e i fondamenti assiomatici*

[9] Toth, *Aristotele e i fondamenti assiomatici*, p. 617.

della geometria, considered your masterpiece[10]. How did you discover the presence of non-Euclidean fragments in Aristotle's works?

Toth: At first I read Aristotle in translation and in *de Caelo* I came across the theorem: "It is impossible, for instance, on a certain hypothesis that the triangle should have its angles equal to two right angles" – (which means, in this theory, that there are no triangles whose three angles are equal to two right angles; the sum of the angles is therefore, in all triangles, different from two right angles, or, all triangles are non-Euclidean) – followed by: "if this is how things stand, then also the diagonal of the square will be commensurable" (*de Caelo*, 281b)[11]. A truly beautiful, amazing non-Euclidean theorem! But I was not bewitched; I wrote: "as is well known, in Aristotle…," because when you write about Aristotle you are writing about the philosopher who is studied and commented on the most, who can boast the most research in his name. Therefore, I did not have the slightest inkling that I had discovered something unknown and new, on the contrary, I was certain that I had read something that was common knowledge, that everyone had known for a long time, except for me who lived in this wretched closed space which Western literature did not penetrate. It took me a long time to realize that these non-Euclidean fragments had simply been ignored by the whole two-thousand-year exegesis of Aristotle.

Das Parallelenproblem was already drafted when the director of the Bucharest National Library, a friend of mine, told me he had seen the title of a book from 1947 by Sir Thomas Heath, *Mathematics in Aristotle*; and, incredibly, he was able to get his hands on it for me (it was not easy to get these books abroad). Heath was the greatest specialist on Greek mathematics, and I expected to find all the fragments I had found quoted and commented on as being common knowledge. But there was nothing, nothing at all. Heath had reproduced some of these fragments, but with no comments of great interest. The only reference that I found interesting was the reference on page 100 to one of the weakest non-Euclidean fragments contained in the *Physics*, with an exclamation that said: "It is not possible that Aristotle could consciously have conceived such an idea as Riemann's. […] It is as if he had had a sort of prophetic idea of some geometry based on other than Euclidean principles, such as the modern 'non-Euclidean' geometries." Heath, moreover, added: "But this is impossible!" and he did not even go on to mention any of the other

[10] The first article on the subject is "Aristotle and non-Euclidean Geometry" (in Hungarian), *Korunk* (1966), pp. 844-852. The work on parallels appeared in *Archive for History of Exact Sciences* (1967), pp. 249-422; the third work is *Aristotele e i fondamenti assiomatici della geometria*.

[11] See I. Toth, *No! Palimpseste. Propros avant un triangle*, Paris: Presses Universitaires de France, 2000, p. 252.

numerous, by far more significant, non-Euclidean fragments. It was at that point that I realized that these passages in Aristotle had until then effectively been ignored and I began a more systematic archaeological study of Aristotle's petrified *corpus*. To tell the truth, it was not difficult, everything was floating on the visible surface of the text. However, it was in the exegetic work of immersing myself in Greek spirituality, and above all making my interpretations in relation to a system of reference of mathematical thought, that I met serious difficulties. It was hard and complex work but in the end I got the impression that those scattered fragments were part of a single and connected whole: a singular Greek vase, madly eccentric and unique due to the monstrous non-Euclidean figures tattooed onto its surface.

A well-known French philologist, Charles Mugler, wrote a book on Plato[12] and came to the conjectural conclusion that there were already non-Euclidean reflections in the Academy community. But Mugler made no reference to the non-Euclidean fragments in Aristotle. His thesis was violently attacked by the famous classical scholar Harold Cherniss, who wondered how Mugler dared to conjecture about non-Euclidean geometry in Plato's Academy without proof from any texts. For me it is still a mystery how Mugler was unaware of the non-Euclidean texts in the *Corpus Aristotelicum*. By contrast, one hundred and twenty years ago, by exclusively analyzing the first book of Euclid's *Elements*, Charles S. Peirce, the great American logician, had already been led to the categorical conclusion that the Euclidean text implies manifestly non-Euclidean reflections underlying its preparation.

In 1981, when I was lucky enough to share a year of study with Cherniss at the Princeton Institute for Advanced Study, he generously offered me crucial help in deciphering and correctly reading the non-Euclidean fragment in *de Caelo*, contained in the manuscript of the *codex Vindobonensis*. On that occasion I naturally asked him his opinion on the extravagant contents of the text but, however determined, he stuck to a laconic answer, avoiding all concrete detail: "This must be taken very seriously."

At best, the world of historians of science and philologists regarded the product of my Aristotelian archaeology with great skepticism, at times with accusations of "madness." Only a minority – a negligible but perhaps homeopathic quantity – greeted my excavations with positive interest, some even with enthusiasm. Among the philologists, I recall Kurt von Fritz, the great historian of Greek thought, Euanghelos Stamatis, the publisher of Euclid, Lorenzo Minio Paluello, the great specialist in the texts of Aristotle, Giovanni Reale of course, and also Helmut Flashar, who included my results in the last

[12] Ch. Mugler, *Platon et la recherche mathématique de son époque*, Strasbourg–Zürich: P. H. Heitz, 1948, anast. reprint Naarden:Van Behkoven, 1969.

edition (1983) of his *Aristoteles*, the third volume of the classic and prestigious *Geschichte der Philosophie* edited by Ueberweg; among the historians and philosophers I want to mention B. L. Van der Waerden, Willy Hartner, Marshall Clagett, Thomas Kuhn, Sir Karl Popper, Ludovico Geymonat, Ferdinand Gonseth, Jules Vuillemin, Adolf Yuchkevitch, Izabella Bachmakova, Boris Rozenfeld and – last but not least – Hans Freudenthal, who discovered, at the same time as I did, the non-Euclidean nature of one of these fragments; his work, which he sent me the manuscript of in 1968, was not published until after his death in 1991. But I fully realize this is indeed a small minority. Nevertheless, I must confess that I've had a vague feeling recently that the wind is shifting. We'll have to wait and see…

Polizzi: You have shown that the mathematical discussion on foundations, very much present in Plato's Academy and taken up again by Aristotle, also took into consideration the hypothesis of the existence of non-Euclidean axioms and how this was combined with an investigation into ethics. Unlike Plato, who gave Euclidean geometry an ontological foundation in mathematical Ideas, Aristotle considered the decision on choosing between Euclidean and non-Euclidean axioms a free choice and decision; and it is not fortuitous that he gave a geometrical example in his *Eudemian Ethics*, to attest to man's freedom to choose between the good (Euclidean geometry) and the bad (non-Euclidean geometry).

Toth: In fact, I found it absolutely incredible that the only example Aristotle gave in detail to illustrate the subject's freedom to choose one of the two extremes of an open alternative, which could not be decided by way of logical reasoning, his only example of reference, is the alternative between a Euclidean triangle and a non-Euclidean one. In this text he also mentions – in the same proposition, together with the common, Euclidean square – the eccentric figure of a non-Euclidean square whose corners are flat, as each are equal to two right angles. They are evidently improper angles because their adjacent sides lie on the same line, a line that is also identical to the perimeter of a square. It is the maximum square of a non-Euclidean plane, its perimeter is a single finite line, closed on itself like a circle. But even though it has a center, just like a circle, it is not a circle, because the circle's only point of contact is with its tangent, while the tangent touches the perimeter of the square at all points. None of the many Aristotle exegetes has ever dwelled on these two millenary steps to comment on this truly monstrous yet infallibly correct figure. What is more, Aristotle concludes this long passage in the *Eudemian Ethics* with a very personal observation, something that is extremely rare in his works: *at this moment we can say nothing more, about these things, than what we have said, but we cannot remain silent about them.*

It seems that my reflections on the role of the subject's freedom at the basis of mathematics began with Aristotle. Little by little I ended up realizing that the implicit and most profound sense of these passages in Aristotle's *Ethics* lies in the message that *Ethos is above Logos*, that the freedom of the subject is the ontic foundation of mathematical being. It is in Aristotle that mathematical thought makes its first appearance on the horizon of freedom. These passages from his *Ethics* already include the secret idea that the ontic foundation of geometry is to be found within the subject – the transcendental subject – of mathematics. The secret was to be made public in Descartes' *Méditations métaphysiques* and in his correspondence with Father Mersenne: the Euclidean theorem that states that *the triangle should have its angles equal to two right angles* is true because and only because in his freedom and absolute omnipotence God wanted it to be and made it true; because in the end in geometry it is *God* – the transcendental subject – who is the ultimate *source of truth*, *deus fons veritatis*.

The presence of the transcendental subject, as the foundation of geometry, is more explicit in one of Aristotle's *Problems* (XXX, 7). In fact, in this text we read that we would feel the same pleasure – *hedone* – if the sum of the internal angles of the triangle were equal to two right angles as we would feel if they were not. An absolutely tremendous statement that we will not find again until Gauss, Bolyai and Lobacevskij! In connection with future contingents, he gives the example of the Salamis naval battle, recalled in his *de Interpretatione* (IX). Tomorrow there will be a naval battle and there will be a winner and a loser. But, in contrast to the geometrical alternative, we would find it pleasurable if and only if the winner is our fleet and never if the winner is our adversary. The metaphor is significant because it compares the geometrical alternative decision to an as yet undecided naval battle. The reason for the metaphor is obviously that neither can be decided by logical means and that, in both cases, their decision issues from the space of subjectivity and is the work of the subject's freedom. The fundamental difference lies in the fact that in the event of a *naumachia*, we only feel joy in the event of our own victory, while in the event of a geometrical *naumachia* we would feel impartially the same joy both in the event of non-Euclidean and Euclidean victory. Evidently, here the author is speaking of the geometrical alternative as an undecided alternative and saying that, like the *naumachia* of Salamis, it is undecidable without the intervention of the subject.

Polizzi: Remaining on the topic of Greek mathematics, you have paid a great deal of attention to the paradoxes of Zeno of Elea, as part of a philosophical interest that leads from Aristotle to Russell and Bergson. Your first work on the subject is from 1969 and it dwells on the theme of actual infinity. But there are other investigations into paradoxes, like the one from 1979 that distinguishes the Dichotomy paradox from the Achilles paradox[13].

Toth: I started off by very closely reading the classic *Dichotomy* and the so-called *Achilles* texts, handed down with great precision and loyalty by Aristotle in his *Physics*. It was not difficult to realize that they are two radically different texts. In *Dichotomy* the word *telos* appears explicitly, determining a given, pre-existing limit, and it deals with just one moving object – *That-which-is-in-motion* (*to pheromenon*) – that never arrives at its *telos* because the axiomatic premise to the topic forces it to be at the halfway stage – *midway in the journey* – of the remaining distance separating it from the *telos* at every step. The fundamental relationship in the *Dichotomy* is the ternary relationship "within" that defines "halfway." Contrary to the *Dichotomy*, the so-called *Achilles* text does not contain any reference to a *telos*, even implicitly, and, again in formal opposition to the *Dichotomy*, it requires two actors, mentioned in the text as the quickest and the slowest (*to tachiston, to bradytaton*). The tortoise is a later invention by Simplicius. In conformity with the premise to the argument, the quickest chases the slowest, and as a result in every instant he is behind the slowest, and at every instant the two are separated by a finite remainder, increasingly small, but different from zero. The fundamental relationship in *Achilles* is the binary relationship of a "series" in an unbroken chase. The two texts express two radically different problems concerning the domain of infinity. The *Dichotomy* raises the question of whether there is a relationship of equality between an infinite chain of finite segments – covered one after the other by *that-which-is-in-motion* – and the total of the journey to follow, ended by the *telos*. In other words, the question is whether there is or is not a relationship of close *equality* between the limit value and the infinite series converging towards the limit. "You will never be able to arrive at the end of the hour" – to use Spinoza's formulation. On the other hand, *Achilles* concerns whether or not a limit exists – the meeting point between the quickest and the slowest – a limit evidently situated in the transfinite, beyond the endless sequence of finite segments that separate the quickest from the slowest at every step. It is a problem concerning being, in the absolute sense of the word, a purely ontological problem, which is therefore much more serious, more decisive and more sophisticated than the question whether a relationship of equality exists.

Eudoxus, Plato's friend, had already found an extremely ingenious method for demonstrating the existence of a relationship of equality in the case of reasoning such as that in *Dichotomy*; that is, in the case in which the limit, the *telos*,

[13] See *"Ahile." Paradoxele eleate in fenomenologia spiritului*, Bucharest: Editura Stiintifica, 1969, and "Aristote et les paradoxes de Zénon d'Élée," *Eleutheria*, 2 (1979), pp. 304-309.

is given, its existence is guaranteed *a priori* and its arithmetic expression can be included in the text of the discourse. This was, for example, the case of the theorem concerning the volume of a cone equaling one third of the volume of the corresponding cylinder. First of all, the existence of the cone is ensured, its arithmetic expression, "1/3" is an articulated word, a *logos*, which can be inserted without any difficulty in the text of the discourse. Indeed, it is demonstrated, absurdly, that it is impossible for the volume of the cone to be smaller or larger – in a word, unequal to one third of the corresponding cylinder. So, if the axiom of the excluded third is accepted – "either equal or unequal" – the volume of the cone must be equal to one third of the cylinder. The logical "tertium," which is excluded from the axiom, is the simultaneous affirmation of two negations: "neither equal nor unequal" (to one third of the cylinder).

But in the case of reasoning such as that used in Achilles, Eudoxus' method is not applicable. It is precisely such reasoning that we are inevitably led to by so banal – and, apparently, so simple – a problem as measuring the length of the diagonal of a square. We know that the Pythagoreans had already developed a recursive procedure, an infinite algorithm – in his comments on the *Republic*, Proclus calls it the *elegant theorem of the Pythagoreans* (*hoi Pythagoreioi* [...] *theorema glaphyron*) – that produces an infinite series of rational values, *logos*, which alternatively express the measurement, by excess and by defect, and therefore the length, of respectively smaller and larger segments of the diagonal of the square. It is basically a very simple measuring procedure that Aristotle calls *antanairesis*; in current terms it is called the *Euclidean algorithm*. Since this measurement procedure is without end, the result is that the diagonal cannot be measured. In Aristotle's *Prior Analytics* we find an allusion to such a demonstration of the inability to measure the diagonal of the square by means of a Zenonian infinitesimal argument, something that attests to the fact that the link between the two arguments was known at the time. In his *Republic* (546c) Plato defines the *logoi* implied in the antanairetic measuring procedure using the expression *inexpressible diagonal*, *diametros arrhetos*. They are evidently rational numbers, approximations by defect and excess of what Plato defines using the term *expressible diagonal*, *diametros rhetos*. There are therefore two infinite series of *logoi*, of expressible diagonals, one going up, and the other going down (two members of the upward sequence, 1/1 and 7/5, and two members of the downward sequence, 3/2 and 17/12, are cited to the letter in different dialogues by Plato); the two discourses expressed in arithmetic terms of *logoi* converge towards each other by defect and by excess, but there is no limit *logos* that can separate them. It is an *Achilles*-style reasoning: the existence of a limit measurement, associated with the diagonal, a single and indivisible measurement, a One. The upward sequence of *logoi* has no end, it is open to the right, the downward sequence has no beginning, it is open to the left. Between the two series of *logoi*, there is no expressible arithmetic term, no measurable measurement, no *telos*, no end limit that can be included in the arithmetic discourse of the *logoi* and of which it can be said it is neither smaller nor bigger than the diagonal; in the case of measuring a diagonal, Eudoxus' method is therefore not applicable.

Therefore, we are not making any mistake if – like Zeno – we categorically claim that such a limit, in which the quickest can reach the slowest, does not exist at all or, in our example, if we claim that the irrational number √2 does not exist, that is, despite having a well-defined magnitude the diagonal nevertheless does not have a length – or, more precisely, that √2 belongs to the ontic domain of *me on* and is therefore an arithmetic not-being. Between the two sequences of *logoi* there is nothing but an empty hole of being, filled by the *darkness of not-being* (*ten tou me ontos skoteinoteta*; Plato, *Soph*., 254a), because it is in fact the precise place of this not-being that is √2. The metalinguistic symbol "√2," *the square root of 2*, is the name of a non-existent number. It does not denote or refer to anything. It is the symbol of a numeric not-being: it looks in the mirror and sees nothing. Despite this, *not-being has an assured existence, and a nature of its own* (*ten hautou physin echon*; Ibid., 258b), and the subject knows all its properties – real properties of a not-being – which the stenograph of its definition, √2 •√2=2, expresses with absolute exactness. But whether it exists or not, the irrational √2 is still a well-defined, single and indivisible One. By the way, I remember that Leopold Kronecker, one of the greatest mathematicians of the 19th century and one of the founders of modern algebra, never accepted the existence of irrational numbers like √2; but obviously accepted the formal relationship √2 •√2=2, which expresses the fundamental property of this not-being. It is therefore impossible to express the length of the diagonal with a numeric expression, with a *logos*, the name of this length cannot be articulated or expressed, it is a *logos* that it is impossible to include in an arithmetic text composed in terms of *logos* because it has to be an irrational *logos*, an *alogos*. If, therefore, the diagonal were to possess a measurement, it should necessarily be a non-measurable measurement. All oxymorons! also contradictions in terms, all unbearable! Horrendous! But it must also be said that, by the same token, we are not making any mistake if, contrary to Zeno, we categorically claim the actual existence of a point where the quickest and the slowest meet, if, therefore, we claim that the gap that separates the two rational series of *logoi* is filled with being, if, therefore, we claim actual existence for the number, or the irrational *logos* √2, which separates the two series from each other. The two answers – "Yes" and "No" – are equally compatible with the argument. Neither is wrong, neither can be said to be absurd. Zeno's argument therefore is not absurd, as is frequently repeated. The two answers are therefore equally compatible with the premises, yet neither of the two claims is logically derivable from it, neither is the necessary consequence of the premises: neither is derivable, neither is refutable. Neither demonstrable nor refutable – which means that undecidable and undecidability is equivalent to the invalidation of the axiom of the excluded middle, of the joint affirmation of two negations: neither one nor its opposite; neither "Yes" nor "No"; neither equality nor inequality; the negation of existence and the simultaneous negation of the non-existence of the limit, of a meeting point between the slowest and the quickest. Undecidability with the means of logical inference per force means that a subject is needed to decide upon the alternative. The axiom of the excluded middle is the fundamental axiom of the subject because it contains the imperative to act, to make up one's mind, to necessarily decide on either one or the other of the two terms of the alternative that is undecidable with the means of logic. The subject is non-geometrical space filled by the substance of freedom. The subject's freedom lies in the act of deciding on one or the other of the two terms, in an open alternative. Such a free decision is equivalent to an axiomatic assertion, which can neither be proven nor confuted using logical means. In his freedom, Zeno chose to categorically deny the existence of a meeting point between the slowest and the quickest – without doubt a shocking answer, but one which is also irrefutable, and is indeed an axiomatic statement.

Zeno's reasoning is totally correct, and of an accuracy, elegance and simplicity – but also a subtlety and depth – that are the privilege and the singularity of ancient Greek thought. Although it is unacceptable for good common sense, Zeno's axiomatic answer does not pose any problem for logical reasoning. Indeed, at every step the moving object finds itself behind the *telos* due to the same infinite series of destinations. And with every step the quickest is still separated from the slowest by a gap, which gets smaller and smaller but, for this very reason, remains a finite length. In these conditions, is Zeno's claim not, in fact, a banality rather than a paradox?

By no means! The axiomatic claim opposing Zeno's answer concerning the existence of a limit, the existence of a single *One* which is, in this case, the meeting point between the quickest and the slowest, fits in perfectly with good sense and empirical experience and yet, from a logical and ontological point of view, it contains unsuspected difficulties, of extreme complexity and sophistication which, at bottom, are absolutely unacceptable for good common sense and incompatible with the axioms of solid logical reasoning. Indeed, the place of this limit, the One, is necessarily found beyond the open and endless world of the sequence of *logoi* – ranging as far as the domain of being – and is, therefore, found in the transfinite domain of not-being.

We do not immediately realize that the axiomatic claim of the existence – in the transfinite – of a One, a limit point, where the quickest reaches the slowest, opens up the gates of hell for logic as well as for ontology, and we cannot help but advise everyone not to venture there. It was Plato who opened these gates and for a long time he was the only one who immersed himself in the infernal negative ontology of the irrational; no geometrician followed him. It was Plato who realized the fact that accepting the existence of the irrational implies accepting actual infinity. According to Aristotle's testimony, Plato indeed accepted what Aristotle himself called *actual infinity* and had stubbornly rejected as an evident absurdity. In Aristotle's opinion, Plato introduced the idea and terminology for what he himself called the *indefinite dyad and the One*, essentially an equivalent term for actual infinity.

Polizzi: Lots of research is published on Zeno and it is normal to attempt to interpret him in new ways, an example being the use of quantum mechanics.

Toth: As a matter of fact, all these reasonings that introduce discrete space and time, and therefore indivisible magnitudes, formally contradict the premises to Zeno's two arguments, premises that postulate infinite divisibility and exclude the existence of quantified, indivisible spatial and temporal magnitudes. By quantifying space and time we commit an *ignoratio elenchi:* under the same name, "Zeno," we speak of something totally different, which has nothing to do with Zeno. And above all, the moving object's arrival at the *telos* does not require any demonstration either in terms of quantity or logic or mathematics; Zeno's conclusion must not and cannot be rejected, it does not concern physical movement in any way, the author of *de Lineis Insecabilibus* speaks, in connection with Zeno, explicitly of a movement of thought (*he tes dianoias kinesis*; [Arist.], *de Lin. Insec.*, 968a). Might Zeno's conclusion contradict our empirical observations! Goodness me! It is not the only mathematical assertion that is incompatible with empirics! His necessary premise, the infinite divisibility of a segment of a straight line, is as incompatible with empirical experience and quantum mechanics as the assertion that the *telos* cannot be reached. And, in the end, what is rare, surprising, even mysterious, is rather when mathematical thought and empirics agree.

Polizzi: Let us dwell on the mathematical problem of infinity that you believe emerged with Greek scientific and philosophical thought. On one hand, Aristotle employs a *reductio ad absurdum* to demonstrate there is no number corresponding to $\sqrt{2}$ and condemns any conception of an irrational and inexpressible number as folly; on the other hand, Plato introduces the indefinite dyad as a model of the oscillation of rational numbers, and therefore of *logoi*, thus allowing a close rational approximation of that borderline number expressed by $\sqrt{2}$. You showed that what emerges from Plato's challenge (also found in the *Parmenides*) is that "ontic turnaround" which, by resolving not-being (the "irrational" number that corresponds to $\sqrt{2}$) in being, permits the construction of infinitesimal mathematics in the modern age. At the roots of Greek mathematical thought we can find that ontological fracture, that turning of not-being into being that has been the life blood of all innovative mathematical theories. Would you like to outline the historical itinerary that makes Plato and the Pythagoreans our contemporaries?

Toth: The theory of the indefinite dyad and of the One was certainly part of what are called Plato's "unwritten teachings," as Aristotle tells us in his *Physics*. This has been shown by the works of Konrad Gaiser, Hans Krämer and Giovanni Reale. In fact, Plato did not devote any of his dialogues to explicitly presenting his conception. We are informed of it in part through the summary, quite frankly bad and in part purposefully distorted, given by Aristotle in the last two books of his *Metaphysics*. But in several of Plato's dialogues, such as the *Statesman*, the *Meno*, the *Philebus*, the *Epinomis*, the *Sophist* and above all perhaps the *Parmenides*, we find very important references to the indefinite dyad and a full and painstaking discussion of its ontological status and logical consequences.

The two upward and downward sequences produced by the *elegant theorem of the Pythagoreans*, which I mentioned earlier, are the two members of the indefinite dyad; the *One* associated to this dyad is the very gap that separates them, or, when required, the irrational value $\sqrt{2}$. In fact, this arithmetic object is a One, unique and indivisible, but, even if there is a not-being, its place is defined with absolute precision by its relationship of equality with the actually infinite group of the smallest and the greatest; evidently, that this equality exists cannot be demonstrated; but at the same time, it is irrefutable. His claim is not the expression of a matter of fact. In the last book of his *Metaphysics*, Aristotle exclaims – angered – that *it is the equalization (isasthenai) of the unequal, nearly a One of the unequal; this is absurd and conflicts both with itself and with the probabilities, and we seem to see in it Simonides "long rigmarole" for the long rigmarole comes into play, like those of slaves, when men have nothing sound to say. And the very elements – the great and the small – seem to cry out against the violence that is done to them!* (Arist., *Metaph.*, N, 1091a). And in the *Parmenides*, Plato himself speaks of a phantom of equality (*phantasma isotetos*; Plato, *Parm.*, 165a). Through a formal negation, the inequality of the two indefinite terms of the dyad is suddenly overturned, they are defined as equal with respect to the irrational $\sqrt{2}$; the irrational goes from its initial ontic state of not-being to the positive ontic state of being. The procedure for measuring the length of the diagonal in a square therefore generates an indefinite dyad.

At the end of the 19th century, Plato's indefinite dyad – now denominated the *Dedekind cut*[14] – was to become the very foundation of the modern theory of irrationality. A leap from *me on* to *on*, from not-being to being. Dedekind consciously let himself be inspired by ancient sources and in this connection he spoke of a creative operation, saying that irrational numbers are purely a *creation of the human spirit*. It is evidently a huge ontological break, which categorically contradicts the logic and ontology of the *father of us all, Parmenides the Great*. In the superb pages of the *Sophist* (238a-241e, 258a-260e) there is ample discussion of this extravagant ontology and Plato openly admits that such a change in the subject from not-being to being unquestionably seems to enter the sphere of witchcraft and dishonest black magic (*goes*; Ibid., 235a; 241b), but the Stranger from Elea insists that, despite appearances, the dialectic game of being and not-being is by no means a Sophist joke or an eristic exercise, but a real phenomenon of the spirit that needs to be taken very seriously (Ibid., 237b-c). Georg Cantor himself, the great founder of the modern theory of infinite groups, spoke in one of his works of a *second principle of generation*, of the *dialectical generation of new, freely created concepts*. The indefinite dyad is thus raised to the rank of the only guarantor that the irrational, like √2, really exists and that it is actually equal to the indefinite dyad. It is evidently an axiomatic act: it cannot be demonstrated or confuted. In order to assign being to not-being we require a free subject, capable of making a decision when faced with the alternative formulated by Parmenides himself: *to be or not to be*. But it is not the *Prince of Denmark* of Shakespeare's tragedy, in this drama *On Being* that, according to its subtitle, is the *Sophist*. It is, rather, this mysterious character of the *many-headed Sophist*, – *ho polykephalos sophistes*, who *has compelled us, quite against our will, to admit the existence of not-being* – states, against Parmenides the Great, his admirer, the Stranger from Elea (Ibid., 240c). The constriction exercised by the many-headed Sophist is equivalent to a free, non-refutable [or rather: non-confutable] but also non-demonstrable axiomatic proposition, which establishes the existence of irrational magnitude directly, therefore without mediation of any kind from empirical or logical proof, assigning being to not-being. The axiomatic proposition is the autonomous act of the free

[14] Here is a definition of the Dedekind cut by the author himself: "Whenever, then, we have to do with a cut (A_1, A_2) produced by no rational number, we create a new, an *irrational* number a, which we regard as completely defined by this cut (A_1, A_2); we shall say that the number a corresponds to this cut, or that it produces this cut. From now on, therefore, to every definite cut there corresponds a definite rational or irrational number, and we regard two numbers as *different* or *unequal* always and only when they correspond to essentially different cuts." See R. Dedekind, *Stetigkeit und irrationale Zahlen*, Braunschweig: Vieweg, 1872; ["Continuity and Irrational Numbers," in Dedekind, *Essays on the Theory of Numbers*, trans. W. W. Beman, La Salle, IL: The Open Court Publishing Co., 1948, p. 15].

subject. The *Sophist* reminds us that through the onomastic act of articulating the name that defines it, not-being acquires the ontic state of being, the *being-known*, whose real existence is guaranteed only by its knowledge and which dwells in the intimacy of the subject, in the pure domain of knowledge. Plato also admits that his conception is a sort of spiritual *parricide*, a parricide against Eleatic ontology. The transition from not-being to being is, in fact, the myth of original sin of specifically Western mathematical thought.

The dazzling style of the dialogue creates the atmosphere of a hidden mystery and solemnly exalts the idea of the ontological event of the irrational. The same almost ecstatic enthusiasm is also given off in the dialogues in all the places where the discourse touches on infinity, limits and the right measurement of an non-measurable magnitude, especially in the *Philebus* but also in the *Parmenides*, the *Meno*, the *Statesman*, and the *Laws*.

The exaltation reaches fever pitch in a passage from the *Epinomis* (990d) – a passage explicitly devoted to the irrational *number*. It deals with the transformation undergone (using the catalyst of an elementary geometrical construction) by a *number* (*arithmos*) by nature (*physei*) not square (such as $2=1\bullet 2$ for example, which cannot be broken down into equal factors) into a square number (the very number 2, therefore, broken down into two equal factors: $2=\sqrt{2}\bullet\sqrt{2}$). The discourse trembles with true mysticism: *and this will be clearly seen by him who is able to understand it* (*to dynameno synnoein*) *to be a marvel not of human* (*thauma ouk anthropinon*), *but of divine origin* (*gegonos theion*). A long time before Plato, the Pythagorean Lysis equated the irrational and inexpressible number with divine being (*arithmos arrhetos ho theos*) despite widespread opinion to the contrary, in fact there is no serious reason to dispute the authenticity of Athenagoras' account). So, according to a celebrated account of Aristotle's (*Metaph.*, A, 983a 16), the irrational aroused the impression of a phenomenon that comes within the domain of the miraculous (*thaumaston*), especially among those with no knowledge of geometry. Finally, we have a later gloss annexed to a famous codex of the tenth book of Euclid's *Elements*, also transmitted by Pappus of Alexandria, in which the event of the irrational is presented in the form of a true mythological tale. With the same tone of mystic exaltation, this myth of the irrational contains a message of terror, perdition and destruction. According to a tale told by the Pythagoreans (*Pythagoreion logos*), the one who first released his theory from its hiding place and made it public perished in a shipwreck. And the text ends with a discourse cloaked in almost sibylline mystery, a sort of evil curse addressed to those who divulge what should remain hidden so that their soul may never find rest: *Hither, thither tossed by adverse waves, Upon a shoreless sea, they blinded roll, Unable to resist or to the tempest yield.* From these words we can clearly perceive the thundering threat of the punishment aimed at the authors of the parricide, the "friends of the Ideas" who assign being to not-being. It is a confession and a historic testimony to the deep intimate conflict of the spirit with itself, aroused by the awareness that accepting the irrational results in being having to be assigned to not-being.

The greatness of Plato's mathematics lies precisely in the fact that he included not-being, the *alogos*, in the universe of being, of the *logos*, and that he built a mathematical rationality that included the irrational without clashing with the problem of its ontological consistency. The sudden meeting in Plato's work of these two great lines of thought was an immense, decisive event of the Spirit: philosophical speculation and mathematical reasoning. Its result is the emergence of a new domain of being, the transcendent domain of mathematical infinity, which these days has become the dominant object of mathematical knowledge.

Polizzi: The Greeks discovered the existence of a "non-measurable measure" and an "irrational reason." A beautiful book of yours – *Lo schiavo di Menone* – is dedicated to giving a "dramatic" description of this discovery so difficult to understand without departing from a schematic and canonical vision of reason, while at the same time weaving a tight commentary on the *Meno* 82b-86c.

Toth: The book on the *Meno* came about thanks to Giovanni Reale. Reale had to edit the *Meno* and asked me if I could make a comment on the mathematical passages, on the famous discussion between Socrates and Meno's young slave. The dialogue has nothing to do with any claim to be a Socratic method of teaching geometry, as is often presumed. In reality, the dramaturgy of the *Meno* is the proof of the real drama of the Irrational, of the upsetting dramatic event which resulted in the *Alogon* penetrating the universe of absolute *Logos*. More concretely, it stages the process of the *generation into being of the just measure* (*Pol.*, 284c – in particular of the diagonal of a square) through *a fusion of the unlimited infinite and of limit* (*genesin eis ousian* […] *tou peratos* […] *metron*), by introducing a *number* (*arithmos*) – a process in which the infinite is seen as a One (*tou apeirou […] eis hen*; *Phil.*, 25a-26d). Allow me to emphasize, by the way, that these passages in the *Philebus* are quoted by Georg Cantor in his work founding the new concept of irrational numbers, in favor of his revolutionary theory of infinite groups.[15] The *just measure* and the *number* that Plato speaks of in the *Statesman* and in the *Philebus* can evidently be nothing but a *non-measurable measure* and an irrational number, inexpressible in terms of *Logos*. The single object, the One that generates the infinite algorithm, created by Meno's slave, is precisely the indefinite dyad that defines the irra-

[15] See G. Cantor, *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts,* edited by E. Zermelo, Berlin 1932, reprint, Hildesheim: Olms 1962, p. 204.

tional and non-measurable measure of the diagonal of a square. Knowledge of the irrational is not the product of a discovery, and even less so of a demonstration; its appearance is immediate, acting without any mediation, like an instantaneous revelation, a true bolt of lightning – the paralyzing electric shock of a torpedo that plunges the subject into the hypostasis of the young slave, in his astonishment, consternation and terror. It is not a "discovery" but an act of becoming aware of the inexpressibility and therefore the irrationality of measuring the side of a double square. It is that delicate spiritual phenomenon that Plato calls *anamnesis*, which is staged in the *Meno* like a true drama lived by the spirit, whose role has been given to Meno's young slave.

The most astonishing analysis of the indefinite dyad and of generating the irrational *One* is narrated in the *Parmenides* in the form of a metaphor that is absolutely incredible in its mathematical geniality and depth of speculation: the metaphor of having the same age, of the Eldest and the Youngest (*to presbyteron*, *to neoteron*) being of the same age (*ten hauten elikian echein*). It is evidently an *Achilles*-style reasoning, preceded by a presentation of the impossibility of exactly measuring the length of the diagonal of the square using the infinitesimal procedure (Plato, *Parm.*, 140b) introduced in the *Meno*.

The question examined in the *Parmenides* is the existence of a time when the Youngest will have the same age as the Eldest. The question seems to be absurd due to the unheard-of banality of the answer which springs immediately to everyone's mind: no! no of course not! Since the difference in age between the Eldest and the Youngest remains unchanged and constant for all eternity. Yes, of course, the difference in age always remains unchanged, but in the *Parmenides* it is not a question of the difference in ages, but the differences in the two speeds of ageing. Indeed, the Youngest, in pursuit of the Eldest, ages more quickly than the Eldest and, in turn, the Eldest ages more slowly than the Youngest who is following him. And so once again we have the Slowest being chased by the Quickest, the so-called argument of *Achilles*! Therefore, the spatial translation is replaced in the *Parmenides* by the diachronic process of ageing, by movement in the temporal dimension. The two figures are in permanent motion in time even if they stay still in a fixed point of space. But in Plato it is not a matter of two people, the youngest of whom pursues the eldest in age (it seems necessary to note that the two nouns are quoted in their neutral gender "*to*," corresponding to "das" in German), and, indeed, contrary to space, no movement of translation is possible in time; therefore, we are speaking of the immanent flow of time in-itself, of the movement of the instant itself, *to nyn*, of the young present running towards its future, towards an older present. The young and rapid present of positive time, of common time that flows from the past towards the present, from the present towards the future – it is the young time whose flow is identical to ageing – thus pursues itself in the hypostasis of its own future present, the young, present and faster instant pursues the older and slower instant of tomorrow, evidently without being able to achieve the relationship of having the same age.

Transplanting the spatial chase of the slowest by the quickest to a temporal setting, simultaneously substituting the two runners separated by space with a single and indivisible diachronic whole, the *Instant*, whose components, the young present and the old future, are the two simultaneous hypostases of the same and only indivisible *One* – think, if you prefer, of the monodic concept of the *complex number* – constitutes, indubitably, a spiritual performance without precedent, an extraordinary intellectual pirouette, worthy of unlimited admiration. In fact, unlike in spatial dimensions, it is totally impossible to move along the dimension of time: we are all irremovably fixed to a single and identical immobile point of present time. In the end, the new formulation of the chase in diachronic terms of ageing and rejuvenation means transplanting the chase from the more banal domain of spatial translation to the wholly impossible and absurd domain of temporal existence. But that's only the half of it.

In fact, Plato introduces – in achronological and absolute simultaneity with the ageing of the younger man – a dual movement, the rejuvenation of the eldest, and the inverse chase. The first chase represents positive common time, its present is the young instant, its movement is ageing; the dual and simultaneous chase evidently represents negative time, the present hour of negative time is the old instant, its movement is rejuvenation. Positive time flows from the young present, ageing more quickly, towards the present-to-come, which ages slowly; negative time flows from the old present, rejuvenating quickly, in pursuit of the past present, which rejuvenates slowly. The succession of ages of the young present of positive time goes upwards; the youth – in effect the present – of positive time becomes older and older. The succession of ages of the old present of negative time goes downwards: the elder, the present of negative time, becomes younger and younger.

The decisive point of Plato's reasoning lies in the introduction of a diachronic dual chase comprising two terms that are both infinite: one is that of positive and young time, which ages more and more slowly, deprived of a final closing element, and therefore open to the right; the other is that of a negative and old time, which rejuvenates more and more slowly, deprived of a first element, and therefore open towards the left. It is evidently an indefinite dyad, the One that it defines and that is assigned to it is the single age of having the same age, it is the place in which the Youngest of positive time reaches the Eldest of negative time. Yes, but this place is situated in transfinite, beyond positive or negative time, in the territory of the transcendent. It is a hole, a gap in being. This results in the direct violation of the excluded middle, which Plato formulates explicitly and in a great variety of forms (in my book I quote all these passages;[16] this idea is expressed most clearly in the daring expression: *neither equal nor unequal* (*oute ison oute anison*; *Parm.*, 140b), but its supreme, most radical articulation lies in the following concise formula: *neither being, nor not-being* (*oute esti oute ouk esti*; Ibid., 157a–e). The Youngest's pursuit of the Eldest together with Eldest's simultaneous pursuit of the Youngest is therefore a perfect model of the indefinite dyad. We can also assign to the increasing ages of the Youngest and the decreasing ages of the Eldest the rational values obtained from the *expressible diagonals*, which I mentioned earlier, and then we get the indefinite dyad that defines a One identical to the irrational value $\sqrt{2}$. The indefinite dyad guarantees at the same time the current existence of this limit value; it requires, accordingly, an act of movement outside time, the ontic overturning of not-being into being: the *Instantaneous* (*to exaiphnes*) – *an event of truly miraculous nature* (*physis atopos*; Ibid., 156d).

Plato's profoundest contribution, which proved at the end of the 19th century to be the most decisive for mathematical thought, was that of having recognized the independent existence of the irrational; thus, the irrational "$\sqrt{2}$" has an independent being-in-itself, and, in order to be, it does not need the geometrical figure of the square and its diagonal. It is not at all necessary to consider it the expression of the length of the diagonal, it is not the geometrical figure that guarantees its existence but the indefinite dyad of two terms, two infinite sequences of autonomous *logoi*, without any geometrical reference. The irrational is thus established as a being-in-itself, a new, autonomous and absolute ontic domain. Therefore, in Platonic terms, it can be said that an irrational such as "$\sqrt{2}$" has a *separate* being (*kechorismenon*), totally independent of any geometrical object such as the diagonal of the square; this, it must be said, has played a decisive role in the purely heuristic process that led thought to its knowledge, without forming the necessary condition of its being.

Plato's argument in support of the dual pursuit thesis is purely linguistic. In Greek, the verb "to age" is a reflexive verb, one says "I have got older myself," "I have aged myself." This syntactical detail is wholly sufficient according to Plato to postulate a dual and simultaneous pursuit: "that which becomes older than itself, also becomes at the same time younger than itself" (*to ara presbyteron eautou gignomenon ananke kai neoteron ama eautou gignesthai*; Ibid., 141c). Otto Apelt, the great philologist who restored to the *Parmenides* its long-disputed authenticity, despite his recognizing the *Parmenides* as a Platonic text, could not understand how Plato, this great thinker, could stoop to such bad jokes and ordinary sophisms: so why not admit that the river that flows from its source to the sea simultaneously goes back up from the sea to its source? – Apelt exclaims.

[16] I. Toth, *I paradossi di Zenone nel "Parmenide" di Platone* (1994), Naples: Bibliopolis, 2006.

Polizzi: Along with the *Parmenides* your pupil Vittorio Hösle gave a very original reading of the *Cratylus*.

Toth: In the Academy they discussed non-Euclidean geometry; in the *Posterior Analytics* Aristotle calls it *ungeometrical geometry*. Hösle, who worked with me, demonstrated that in the *Cratylus* there is an important passage (Plato, *Crat.*, 436a–e), obviously concerning the already rather considerable chain of non-Euclidean theorems,[17] that underlines that these non-Euclidean theorems, though all false, from a logical point of view are nevertheless perfectly coherent amongst themselves (*symphonein*, *homologein*); it is also very significant that Cratylus himself explicitly articulates the principle according to which logical coherence is *the supreme indication of truth* (*megiston* […] *tekmerion* […] *tes aletheias*; Ibid., 436c). I had told him that I had always meditated on this passage in the *Cratylus* but I had never managed to discover anything of importance, even though I had a certain suspicion. Hösle understood and interpreted the passage in the *Cratylus* much better than I did.

Polizzi: In short, you gave a revolutionary reading of the mathematical thought in Aristotle and Plato, not just contributing to a radical modification of the vision of Greek mathematics, but above all identifying that close interaction between mathematics and philosophy, the signs of which we no longer manage to grasp because we are now incapable of reading philosophy imbued in mathematical knowledge. What is it that expresses the now irredeemably lost conditions for the unity between mathematics and philosophy in Greek thought?

Toth: "Paradoxes" like those of Zeno are also present in China, but there they have remained a board game, an amusing brainteaser, without any consequences; the Chinese were very astute, very able, truly brilliant, but it seems that they have never had any particular sensitivity for the metaphysical dimension of mathematical thought without practical overtones. Zeno, Plato, Aristotle and the whole tradition that they launched, took one thing that others thought frivolous very seriously, a question for which there was no answer. It was an act that truly founded Western thought: with Zeno's arguments the idea of infinity was installed in the domain of specifically mathematical existence.

Polizzi: In mathematics negative logic was axiomatized by Hilbert in his *Grundlagen der Geometrie*, with his demonstration that it is possible to build a

[17] V. Hösle, *I fondamenti dell'aritmetica e della geometria in Platone*, Milan: Vita e Pensiero, 1994.

complete geometrical system by replacing an axiom with its formal negative. It claimed the epistemological value of the negative, it formed the "postmodern style." Bachelard's "philosophy of no" comes to mind, which hypothesized for the "new scientific spirit" the proliferation of non-Pythagorean arithmetic, non-Euclidean geometry and non-Newtonian physics.[18]

Toth: I read Bachelard's book on the philosophy of no, but I was a bit disappointed, it seems that his reflection sticks to the immediately visible and accessible surface of things. Bachelard stated that modern science had broken with classic science, denying the Cartesian method, and he defended this historic turn. Nevertheless, he went no further than the current situation, he did not follow the deeper value of negative dialectics up to the birth of scientific thought in Greek mathematical culture.

4. Analytical Philosophy versus Mathematical Thought

Polizzi: Your polemic on analytical philosophy is prompted by the philosophy of mathematics and of logic. In a work from 1987 you highlighted Gottlob Frege's logical errors and retraced their origin to the rejection of the main results of modern mathematics.[19] I think we can say that for you Frege is the negative model of mathematical ethics, the exemplification, also on a political level (nationalism, anti-Semitism), of the "reactionary" opposition to the free creativity of mathematicians, and the ethics of mathematical freedom.

Toth: Frege wrote, in the 1903 and 1906 *Jahresbericht der Deutschen Mathematiker-Vereinigung*, the German mathematical association's yearbook, seven polemical, poisonous – and frankly very coarse, vulgar even – essays against non-Euclidean geometry, and against David Hilbert in particular. We can say that the research on Frege, otherwise very rich and prolific, ignored and continues to ignore these works. In his book *Frege. Philosophy of Mathematics,*[20] a truly monumental work, Michael Dummett does not deal with these articles on geometry. For Frege if history is in conflict with logic we must deny the

[18] See G. Bachelard, *La philosophie du non*, Paris: Presses Universitaires de France, 1940; [*The Philosophy of No*, trans. G. C. Waterston, New York: Orion Press, 1968] and Id., *Le nouvel esprit scientifique*, Paris: Librairie Félix, 1934; [*The New Scientific Spirit*, trans. A. Goldhammer, Boston: Beacon Press, 1984].

[19] I. Toth, "Freges mathematische Philosophie und die Mathematik zu Freges Zeit," in G. Jussen (ed.), *Tradition und Innovation*, Bonn 1987, pp. 90–92; Toth also held a seminar on Frege's philosophy of mathematics at the Istituto Italiano per gli Studi Filosofici (Naples, July 1993).

[20] M. Dummett, *Frege. Philosophy of Mathematics*, London: Duckworth, 1991.

former to the point of rejecting non-Euclidean geometry as an expression of substantial irrationality and mysticism. Frege was pitiless with mathematicians who broke the principles of logic. In his opinion, non-Euclidean geometry brings subjectivity into mathematics, subjectivity – in the worst psychological sense of the word – of the empirical subject, of the demographic individual; as if Descartes had never existed, the idea of the transcendental subject remains unknown to him. "Away with non-Euclidean geometry!" he exclaimed; it is a *conjuring trick*, "it must be thrown out (*herausfliegen*) and classified as a mummy together with astrology and alchemy."

As a matter of fact, it was Frege who voluntarily withdrew to the Galapagos islands of mathematical thought, and from there he proffered his sarcastic discourse, full of contempt for everything that produced this mathematics, which was, in his day, still "modern." He hated the very word "modern" and only used it as an invective. Since, according to Frege, "modern" has the same etymology as "*Mode*" ("fashion"), by its very etymology it connotes frivolous and passing adoration and, for him, this *morbus mathematicorum recens* threatened to sink mathematicians into "filth" (*Verunreinigung*). His philosophy imposed a political practice upon mathematicians: it postulated what was allowed and what was not. And all the new procedures of thought introduced by non-Euclidean geometries and Dedekind and Cantor's new theories of irrational numbers were definitely on the black list.

In general there is no link between the political and scientific conceptions of a great scientist. One can be progressive or revolutionary in one field and conservative or reactionary in the other. Due to the remarkable coherence of his personality, Frege was definitely a very rare case: in the deepest etymological sense of the word, he was as retrograde and reactionary in the domain of day-to-day politics as he was in scientific politics. But not even he remained consistent until the end: he was an undisputable revolutionary – in the domain of logic. And yet his relations with mathematical knowledge were a bit unfortunate: all these concrete examples, taken from the domain of mathematics, are wrong; he was also unsuccessful in his tendency to use natural sciences, especially geography, in favor of his philosophical theses. In fact, it is a very bad example: nothing is more radically opposed to mathematical knowledge than geography.

Indeed, when I hear talk of "Frege's mathematical philosophy," I never know what "mathematics" is being talked about. What is, and above all where is, the mathematics that can correspond to the demands of such a philosophy? I am truly sorry, but I have the feeling that, in this context, the term "mathematics" bears no reference. Frege was far from being the only one to reject non-Euclidean geometry. Except for a very restricted group of new generation mathematicians – Klein, Poincaré, Clifford, Helmholtz, Lie, Hilbert, Beltrami, Peano, just over a dozen – the absolute majority of the older mathematicians and philosophers rejected the new geometry. And yet, among the opposition, Frege holds a special and privileged position; not only was he the greatest logician of his time, but he was also the founder of a philosophy of scientific rationalism – the paradigmatic starting point for analytic philosophy – and had an exceptional nose for the logical correctness of reasonings in mathematics and equally in politics. He was a total fanatic of the truth, incorruptible, intransigent and unshakeable. In the conclusion to his political testament – the words to which I prefer not to quote – he recalls the exergue of his life: "Because I want the truth, nothing but the truth." In the case of non-Euclidean geometry, without doubt he was right: the simultaneous truth of two contradictory propositions, E and non-E, is wholly incompatible with logic. It is certainly an undisputable truth. What he unfortunately never learned is that logic is not everything. Not even in the domain of *Reason*, of mathematical *logos*.

I know only one person who can compare with Frege in the strict coherence of his personality: his contemporary William Kingdon Clifford, the greatest English mathematician of his time. At the outset, he was not a geometrician but an algebraist, and also a firm anti-Monarchist, a *republican*, a *radical*, an admirer and follower of Mazzini. He was deeply committed, he held popular conferences on the theory of evolution and on non-Euclidean geometry because, in his view, the new geometry was in agreement with Darwin's conception and brought with it the same emancipatory message of freedom that the Copernican revolution did. He left an enormous amount of work in the field of non-Euclidean geometry, but his interest in the new geometry was motivated by the political side of non-Euclidean thought.

Polizzi: To you non-Euclidean geometry seems to be the touchstone of the limits of analytic philosophy. Despite having taken the formal method of mathematics as the model for philosophical research and making mathematical logic the basis of philosophical language, the analytics did not agree with the non-Euclidean revolution because it would have led to a crisis in the axiomatic rigor and acceptance of a logical dialectic that would have undermined the edifice of axiomatic foundation.

Toth: Analytic philosophy is first of all a total invalidation of classical philosophical speculation: the subject, the Self, are totally eliminated and accused of creating outmoded metaphysical confusion, and non-sense. Everything that classical philosophy, especially the philosophy of Plato, Kant and Hegel produced, is treated with mere sarcasm or ironic quips by the great representatives of analytical philosophy such as Russell and Quine. Quine, for example, designates the concepts developed in Plato in the *Theaetetus*, *Sophist*, *Statesman* and *Parmenides*, his unbearably confusing speculations on being and not-being, with a sort of nickname: "Plato's beard." Russell is no more gentle: he speaks, also in his *History of Western Philosophy*, of the publication of his work *On Denoting* as a revolution, that clears up two millennia of muddle-headedness about "existence," beginning with Plato's *Theaetetus*.[21] But what is worse, in my opinion, is that this current of thought, with its program to transform philosophy into science, uses a method reminiscent of mathematics: formulas, symbolic writing, a mode of expression found in the books of mathematics and mathematical logic and which naturally gives the little-discerning reader the immediate impression of a high level of scientificity. *The book of Nature, yes*, but the book of Man is not written in mathematical language. Mathematical knowledge is the product of Reason, but not of logic, it belongs wholly to the world of Reason but not the universe of logic.

While it is quite widely known that the subject, the Self, is excluded by analytical philosophy, it is less well known that its Achilles' heel is the field of mathematical learning, the field of mathematical knowledge. This may come as a surprise – I realize that – but what I find significant is precisely this: the incapacity of analytical philosophy to explain, to interpret the most important processes of mathematical thought, of mathematical learning. The act of creation shown in the negative, imaginary, irrational number's transition from not-being to being as in the case of non-Euclidean geometries or spaces of more than three dimensions is totally unacceptable for analytical thought; the simultaneity of the contradictory truths, E and non-E, of the domains of opposing beings, a Euclidean and a non-Euclidean universe, each of which containing everything that is – is truly horrific. Two different spaces cannot exist side by side in the same world – Bertrand Russell repeated, and he managed to erase the theory of irrationalism drawn up by Dedekind with an elegant smile of scorn, saying that it had *many advantages,* [*similar to those*] *of theft over honest work*. The solution that he proposes in order to avoid the catastrophe of two spaces with opposing geometries existing side by side is to interpret the words "straight line" in the non-Euclidean propositions as in reality defining a certain curve – perhaps, a certain arc of a circle perpendicular to the circumference of a given circular disk – contained in the absolute

[21] "So – as already indicated in the *History of Western Philosophy* (a work awarded the Nobel Prize for Literature) – I showed how my *Philosophy of Logical Analysis* was able, in one fell swoop, to clear up two millennia of muddle-headedness about '*existence*,' beginning with Plato's *Theaetetus* [...]. Returning to my intentions, I would like to remember that following my work *On Denoting*, mathematical knowledge has lost its aura of mystery," Toth, *No!*, p. 284. See B. Russell, *History of Western Philosophy and its Connection with Political and Social Circumstances from the Earliest Times to the Present Day*, London: Unwin, 1946, and "On Denoting," *Mind*, 14 (1905), pp. 479-493.

model of the non-Euclidean text; so this curve, this absolute geometrical object interprets or represents the word "straight line" in the non-Euclidean propositions. "Absolute model" means a geometrical configuration of the absolute plane, that is, a plane in which the axioms of the parallels E and non-E are undecided and undecidable, neither true nor false. In the absolute plane, the axiom of the excluded middle is not valid for the pair (E, non-E): neither E, nor non-E, this pair does not represent an alternative. The model therefore represents the non-Euclidean universe as its geographical map, its globe on the absolute plane. It represents, evidently, also the absolute text, since this is a real and proper part of both the non-Euclidean and the Euclidean text.

But according to Wanda Szmielew's fundamental theorem, the theorem of representation, it does not matter what model of absolute geometry is *necessarily* a model of either non-Euclidean or Euclidean geometry. The model is an absolute geometrical configuration, its existence is ensured even if there is no non-Euclidean geometry or no Euclidean geometry. And yet its immanent structure has to be either Euclidean or non-Euclidean. In opposition to the universe, in the case of her model, the pair (E, non-E) is strictly subject to the axiom of the excluded third: either E or non-E is a strict alternative. This means that the absolute model is decided in itself and perfectly determined as Euclidean or non-Euclidean; the non-Euclidean nature of the absolute object representing the "straight line" is strictly derivable from the other properties of the absolute model. The Euclidean or non-Euclidean structure is strictly derivable from the other properties of the absolute model. But with the help of the same models we can demonstrate that non-Euclideanism, the non-E axiom, as well as Euclideanism, the Euclidean axiom E, are not derivable from the absolute propositions of geometry. This shows that the non-Euclidean universe cannot be boiled down to being interpreted on an absolute model because the specific non-Euclidean nature of the universe is undecidable, indeterminate and indeterminable, while the specifically non-Euclidean structure of its model is already determined within absolute geometry. Between the universe and its absolute model there is a relationship of isomorphism, of identity of structure, but by no means just any relationship of identity. It is therefore impossible to substitute the non-Euclidean universe where the "straight line" is straight, for its absolute model, where the non-Euclidean "straight line" is curved (e.g. an arc of a circle perpendicular to the circumference of a disk), because such a substitution blatantly contradicts Wanda Szmielew's theorem; it leads, by the same token, to the logical inconsistency of absolute geometry, something that seems to be ignored by the core of the analytical current. In this regard, Willard van Orman Quine wrote that *in the beginning, they were deprived of all interpretation, and therefore of truth. In the meantime they have received serious interpretations and a set of non-*

propositions has been able to be identified with genuine truths. If this were so, then absolute geometry ought to be inconsistent!

But it is not just geometry that opposes the intercosmic universal validity, the uniqueness of truth. Although it has never aroused protests, nor ever generated public scandal, the arithmetic hyperuranium is also dominated by the co-existence in the universe of opposing numbers and different simultaneous truths that contradict one another. Our common arithmetic, of natural numbers, of the sequence 1, 2, 3, …, N, succ. N, …, is based on Peano's axioms. His language is extremely straightforward, it contains two primitive, undefined terms: *number* and *successor*; his axioms seem to be the evidence itself: there is a number, every number has a successor, there is a number that is not a successor of any other number, it is the first in the sequence and it is called *one*, "1"; thus: everything that is a number belongs to this sequence, which is therefore an ontologically closed universe, all the numbers, without exception, are included in it, there are no numbers beyond its confines. This universe is structured in algebraic terms using two operations, two binary compositions. One is an addition and defines the sum of two numbers, its initial axiom is N+1= succ. N: the sum of N and 1 is the successor of N. The other is a multiplication and defines the product of two numbers; its initial axiom is: 1•1=1; as a consequence 1•N=N. In other words, the number "1" is the neutral element of multiplication: when multiplied by "1," every number remains invariably the same. Hence, by starting with these propositions, we can also demonstrate what is called the fundamental theorem of arithmetic or unique factorization: every number can be broken down into a product of prime factors and this breakdown is unique; for example: 30 is equal to the product of the following prime numbers: 2•3•5 – this is the only way that 30 can be broken down into prime factors.

The assertion is evident in itself, but its demonstration is not so simple; it was demonstrated by the ancient Pythagoreans and is found in book IX of Euclid's *Elements*. It is, moreover, one of the reasons for which this universe of numbers is said to be Euclidean. But the deeper reason is that there are also universes of natural numbers broken down by the same numbers 1, 2, 3, …, N, succ. N, … and subject to the same Peano axioms (for example: N+1 = succ. N). With one exception: Peano's axiom 1•1=1 is replaced by the axiom 1•1=2, which, evidently, contradicts it. And what is more, this is not simply a contradiction like that of E and non-E, in itself it is a revolting assertion, a horrible monstrosity. This is immediately followed by the theorem 1•N=N+N; the number "1" is not the neutral element of the multiplication (watch out: N+N cannot be replaced by 2•N). It goes without saying that the last of Peano's axioms remains invariably valid: everything that is a "number" is included within this universe, there are no numbers beyond its confines. The most remarkable trait of this arithmetic is that the Euclidean unique factorization theorem is invalidated in its universe; this is why it is called non-Euclidean. So, the number 30 can be broken down into prime factors in two different ways: 30=1•15 and 30=3•5 (in this arithmetic, 15 is not the same as 3•5 or any other product of two numbers, it is a prime number in the non-Euclidean universe, it cannot be divided either by 1 or by 15). According to oral tradition, this arithmetic was developed at the beginning of the 20th century by the great Italian mathematician Mario Pieri. It is a classic example, cited very briefly in all algebra manuals as an example of a non-Euclidean algebraic structure, but other than that it does not seem to hold much interest. As in the case of geometry, the two universes of numbers, Euclidean and non, each separately contain all the numbers without exception; the two propositions, logically incompatible with each other, are both true at the same time, both are indemonstrable and irrefutable axioms. In fact, with non-Euclidean arithmetic there can be a model inside the Euclidean universe: the group of even numbers that can be interpreted as images or representations of the non-Euclidean numbers: if non-Euclidean arithmetic, based on the axiom 1•1=2, were inconsistent, common arithmetic based on the truth 1•1=1 should also contain a logical contradiction, and vice-versa. However, this does not mean the model can be confused with the non-Euclidean universe of numbers because the successor of an even (Euclidean) number is an odd number that does not represent any non-Euclidean number, while the immediate successor of a non-Euclidean number is a non-Euclidean number which corresponds, as its image, to an even Euclidean number, the successor of the given even number. But there is also a more radical opposition between the universe and its model. In the non-Euclidean universe, the number 30 can be broken down into prime factors in two different ways; in the Euclidean model 60 corresponds to 30 and this, in perfect harmony with the fundamental theorem of arithmetic, can only be broken down into one set of prime factors 2^2•3•5=60, and no other.

The existence of the universe and the truth of the non-Euclidean axiom does not depend in any way on the existence of a model or an interpretation. In fact, non-Euclidean arithmetic can be built in a totally autonomous way, starting from these axioms alone – as in Peano's arithmetic – without taking into account the existence, and even by stating the non-existence of Euclidean arithmetic. In his *Grundlagen der Arithmetik*, from 1884, Frege categorically denied the very possibility of formulating non-Euclidean arithmetic propositions: everything would be destroyed if this happened, thought itself would become impossible. He was clearly wrong. It is true that he was never informed of the existence of non-Euclidean arithmetic – even though such an algebraic structure had already been drawn up a long time before by Dedekind – and it seems to me that the news of the creation, more than a century ago, of such a non-Euclidean arithmetic world has not yet been registered by the otherwise extremely rich analytical literature.

In the 1893 introduction to his *Grundgesetze der Arithmetik*, Frege dialogues at length with the neo-Kantian philosopher Benno Erdmann who seems to have admitted the possibility of other worlds inhabited by other humanoid beings, where other laws of thought than our human ones could be permitted, such as: "one times one is two." Frege's reply is limited to the laconic: "an as yet unheard-of folly." On this point, it is certainly difficult not to agree with him. In any case, the definition that he proposed for the Euclidean number "2," which expresses a property assigned to the concept of "the satellites of the planet Venus" (the extension of this concept to two elements), definitely cannot be applied to Pieri's number "2." But, again in complete agreement with Frege, the problem does not depend on psychology, but on the domain of objective arithmetic learning. And the question one poses is: why on earth has such a psychopathological assertion as 1•1=2 come to be accepted as the foundation of an arithmetic theory as true, objective and eternal, and as universal as the Euclidean truth 1•1=1?

5. Art, Literature and Mathematics

Polizzi: In a recent interdisciplinary convention – *Orfeo e l'Angelo, itinerari dell'etica nella complessità* [Orpheus and the Angel, Itineraries of the Ethical in Complexity] – you presented a paper that also referred to your parallel commitment as an artist (and painter above all), entitled *Arte e matematica: domini di libertà e di creazione* [Art and Mathematics: Domains of Freedom and Creation]. In addition, you had already dedicated a seminar at the Istituto Italiano per gli Studi Filosofici on the relationship between art and mathematics.[22] In what direction did you look for the principle of that correspondence? Is it research into shapes and figures and their creation?

Toth: In the normal, scholastic classification distinguishing the physical sciences and the *technai*, the arts, mathematics is always associated with physics and natural sciences, but in my view the ontology and epistemology of mathematical learning lead us to see it connected with art. I repeat: *Mathesis* and *poiesis* have similar ontological structures.

[22] The convention *Orfeo e l'angelo, itinerari dell'etica nella complessità*, organized by the Casa delle Letterature dell'Assessorato alle Politiche Culturali and by the Dipartimento Cultura del Comune di Roma, was held in Rome on 21-22 March 2001; the seminar, entitled *Poiesis mathesis. Ontologia della matematica e dell'arte* [Poiesis Mathesis. Ontology of Mathematics and of Art], was held at the Istituto Italiano per gli Studi Filosofici in Naples on 4-8 May 1998.

Polizzi: In the postmodern perspective that you consciously make your own, novels possess the exactness of a mathematical theory. This can be seen in their description of a universe, the result of a totally free cosmopoiesis that is in itself the source of truth. In 20th-century art, you refer to exactness as the very configuration of a novel, finding it to hold pure forms of geometry. Could you be more specific about that idea of exactness and give some examples in contemporary literature?

Toth: Mathematics is famous for being an exact science: I am defining a tetrahedron if I say that in the four vertices *ABCD* no vertex is located between two other vertices. Obviously, in real physical space there are three-dimensional solids that correspond to the definition of a tetrahedron. If I now consider five points *ABCDE* and I posit *a priori* the same condition that no vertex be found between two other vertices, I have built a *pentatope*, the simplest figure that defines a four-dimensional space. But where is the *pentatope*? I know it in an equally as exact manner as the tetrahedron, *ABCD*, the text of its definition gives a description of absolute exactness, and from this I can conclude that in this four-dimensional space there are six regular bodies. But contrary to the five regular polyhedrons of three-dimensional physical space, all these six regular polytopes of four-dimensional space are non-beings, bodies that do not exist.

In fact, if we remain faithful to the meaning *of the good old word "exist"* – as Quine demands and, with him, without doubt all those who have any common sense – we have to say that, *in our common sense usage of "exist*," the four-dimensional cube is as much a non-being as Emma Bovary. And yet we know the nature and all the properties of the two with the same absolute exactness, with the same perfect certainty; the two texts offer us an eternal and unchanging knowledge of their object. Had Emma Bovary existed, the novel *Madame Bovary* could have been corrected by journalists or historians on the basis of documents, like a biography of Marie Antoinette. But even if Emma Bovary did not exist, she cannot add anything to this story, she cannot cut anything out, she cannot contradict or correct the author. In this connection, please allow me to quote once again Plato's *Parmenides*: *For that which is said "not to be" is known to be something all the same.* And on my part, I would add: only not-being is knowable with absolute exactness.

Polizzi: We were talking about your research as an artist: in 1997 an exhibition was devoted to you at the Museo Laboratorio d'Arte Contemporanea at the University of Rome organized by Alberto Zanazzo and presented by Maurizio Calvesi, and in your latest book there are some reproductions of your collages[23]. You defined the paper and text collages a characteristic expression of the postmodern. Technically speaking, what is your line of painting and how should your collages be read?

Toth: Maurizio Calvesi asked me the same question. I called these "drawings" *collages métaphysiques*. Art has various sources of inspiration: a beautiful woman, a tree, a horse, a religious feeling, a naval battle, a sunset can all be objects of inspiration for art. Here the source of inspiration is the text, the texts. The collage *La creazione secondo San Tommaso d'Aquino* [The Creation according to Saint Thomas Aquinas] is inspired by Saint Thomas (who said that God cannot produce a rectilinear triangle the sum of whose angles is not equal to two right angles): I expressed a sentimental reaction to this line from *Summa contra gentiles* (2, 25). I cannot give a precise explanation, it is not a figurative depiction, it is not a mimetic or didactic depiction. It is a spontaneous emotional reaction to the words of Saint Thomas, articulated using the means of figurative language. Here is my *Seductio ad absurdum*, in which the harpy of non-Euclidean geometry is torturing Aristotle, with a quote from the *Posterior Analytics*: Which of the two opposing assertions has the *logos* – the truth, the *raison d'être* – of the triangle: the one that says that the sum of its angles is equal to, or the one that says that this sum is not equal to two right angles? The question remains undecided, Aristotle gives no answer, the torture of indecision continues. It is my sentimental reaction; these collages depict the beauty of the texts.

6. Commitment to Philosophy

Polizzi: You have expressed your involvement in the political side of public life through your defense of philosophical studies in Europe. In 1991 you represented the Istituto Italiano per gli Studi Filosofici di Napoli [Italian Institute for Philosophical Studies in Naples] at the European Parliament in Strasbourg and you signed an *Appeal for the defense of philosophy and the extension of its teaching in European secondary schools*. Could you give us a general outline?

Toth: What has happened and what I consider from a certain viewpoint a cultural tragedy that has hit the whole of Europe, the suppression of the teaching of philosophy in certain countries, is the result of a lengthy evolution in

[23] I am referring to the *collages* collected in the catalogue published by A. Zanazzo on occasion of the exhibition presented at the Museo Laboratorio di Arte Contemporanea in Rome on 10-27 November 1997, which Toth illustrates in his answer.

history. Ever since philosophy has existed in a literary form, starting with the Pythagoreans, Plato and Aristotle, the problem of whether philosophizing is of any use, whether this occupation or rather preoccupation has any sense, has continually been posed. They are reflections that have accompanied the whole history of philosophy. It is also part of the specific nature of philosophy that its new ideas and new forms of self-awareness at first seem extremely paradoxical and that at times the words to articulate them don't even exist. Later on, they perhaps come to sound less paradoxical, but a bit strange all the same, and in the end they become totally banal. They become a banality, yes, but this means that once acquired, philosophical knowledge does not fall into oblivion. I will give just one example. If I say today: man is free, this is totally banal. You can find this written in the daily papers or on the lips of any politician from the left or right, everybody knows it, it is repeated *ad nauseam*. Well, let me remind you that only a century and a half ago one could not utter the word "freedom" without running a risk, and one could not claim without risk that man was a free being: you risked your own freedom and even your life. Now, I'd like to add another observation. Curiously enough, the idea that human beings are free made its first appearance in Aristotle's *Ethics*. In the *Great Ethics* and the *Eudemian Ethics* there are some chapters in which human beings are characterized by their freedom. Unfortunately, the word freedom is not uttered by Aristotle because the words "free" and "freedom" – *eleutheros* and *eleutheria* in ancient Greek – only meant the social status of the free man in comparison to a slave. In those chapters Aristotle is like a dumb man making superhuman efforts to express himself. In the lexis at his disposal there is no word to give the idea of this new conception. As I have already mentioned, in order to illustrate his thought, it is very interesting that Aristotle turned to the free decision of the geometric alternative: E or non-E. In those chapters of Aristotle's *Ethics*, human beings attempt for the first time to express the awareness of their freedom; and this was a great effort destined to last for centuries, millennia, until the moment in which that idea would become as banal as it is today. In a word, what I want to say is that without this birth of the idea of freedom as self-knowledge, as the knowledge that human beings are free, without this knowledge, without this becoming aware of freedom, all the achievements in the field of politics, society, art, and the entire emancipation movement would have been impossible. It is precisely the great effort of becoming aware of freedom that was produced in the field of philosophy; within this broad, confused, and to all appearances useless field called philosophy the phenomenon of becoming aware of freedom was produced, without which we wouldn't have anything, either in the sphere of modern art, or in that of mathematics and modern physics, or, above all, in the field of social justice, political life, a political life and a social life better suited to a human life.

Polizzi: Could you tell us the reasons for this both pedagogical and political commitment aimed at European students and teachers of philosophy, could you give us an idea of the new *paideia* that you intend to propose by diffusing philosophy?

Toth: The singular nature of philosophy lies in its object. The object of physics is natural objects, the object of astronomy is the stars, the object of medicine is the human body, but the object of philosophy is the human subject, the human subject as the maker of its own history, of social praxis. What we call philosophy is not a science but it is, all the same, something: a knowledge, the subject's knowledge of the subject. It is the subject's self-consciousness of being a subject. With this statement we are saying that in the field of philosophical speculation this process of becoming aware has taken place, human beings have become conscious of what they are; for example: free. We cannot truly hope to find solutions to the great quantities of problems of the human condition that await us today and in the future if the reflection on the human condition, on human destiny, on the human subject, on human being in general is suppressed in such a way as has been attempted in recent years, precisely in those countries that have produced great philosophical speculation.

7. Non-Euclidean Geometry and Freedom

Polizzi: Freedom of the subject, freedom of Spirit, is expressed in the process of *Aufhebung* that permits geometry to be incorporated into a superior value of truth surpassing, at the same time, Euclidean and non-Euclidean geometry. But also freedom recognizable as a necessary act of social space and historic time. In substantially Hegelian terms, you have understood the process of verification as "a vertical phenomenology of individual consciousness" that assumes its "ontic medium" in the "consciousness of the collective subject of history."[24] To give an example: the conception of non-Euclidean geometry comes about from the individual consciousness of Lobacevskij, Gauss and Bolyai, but it came to be included in history when it was accepted as a value of truth. Through the vicissitudes of geometry, you illustrate the magnificence of a negative ontology: "the enormous power of the negative, the energy of the pure 'I'" constitutes an act of foundation based precisely on not-being and brought to life by a free and dialectic reason.[25] We are talking total Hegelianism!

[24] These are expressions from I. Toth, "Scienza e scienziati nell'età postmoderna. Il valore scientifico e il suo ruolo nella costituzione della scienza," *Intersezioni*, 8 (1988), pp. 311-339.
[25] The famous phrase from the *Phenomenology of Spirit* is used with a clearly Hegelian slant in

Toth: The real reason for this interest in non-Euclidean geometry, for this preoccupation and, ultimately, for the subsequent victory of non-Euclidean geometry must be looked for outside mathematics. Mathematics is not an island, *no man is an island* (John Donne, *Devotions upon Emergent Occasions*, XVII), at all events mathematics is immersed in a spiritual context. Mathematics cannot be isolated because it is in Spirit. Hegel is interested not in mathematics but in human history, he saw the whole of history as gaining awareness of freedom. And this is true. It is very interesting that mathematics became aware of freedom in the shape of non-Euclidean geometry precisely in Hegel's day. There is no possibility of a reciprocal influence. It is very odd, perhaps even very mysterious. The same spirit had been working in both mathematics and philosophy: that is, in gaining awareness of freedom. Mathematics took part in this event and the logic of freedom came to the surface, became visible in this too. In other words, mathematics participated in the phenomenology of freedom.

Polizzi: Your book *No!* is really quite strange. You called it a "palimpsest of words and images," which, in the style of a literary collage, presents an imaginary dialogue involving hundreds of *dramatis personae* (no less than seven hundred characters) engaged in a heated debate on the "scandal" of non-Euclidean geometry outside any demarcation of time and space. You also remark – in the "Afterword" – that yours is a "text on the writing of mathematics," in the same sense attributed to the "text" by the literary theories of Roland Barthes, Walter Benjamin and Michel Foucault.[26] Apart from the initial occasion (dating back to 1976) and the continuous rewritings, what was the structural reason that led you to "rewrite" a palimpsest?

Toth: In *No!* there are more than 700 characters. The form of writing that I used in the palimpsest of *No!* is the same as for my collages, they too are palimpsests, the method is the same. It is a *collage métaphysique* because the source of inspiration is a metaphysical source.

Polizzi: In this book you again put in play, explicitly, the relationship between freedom and truth, as if the laborious affirmation of the non-Euclidean revolution were the best example of the progressive conquest of creative and productive freedom and, thus, of possible truth.

Toth: In fact, all geometry is a cosmology. The "true" non-Euclidean geometry appears at the beginning of the 19th century and is good for nothing; the nega-

Toth, *Aristotele e i fondamenti assiomatici della geometria*, p. 613.
[26] "Afterword and thanks," in Toth, *No!*, p. 458.

tive and imaginary numbers are mysterious but they are useful, this "monster" of thought that is non-Euclidean geometry served no purpose. Non-Euclidean geometry has no motivation, it has nothing to do with double entry, like negative numbers. And then, after around one hundred years, suddenly it is applied in physics, in the theory of relativity. How can that be? The mathematicians cultivate the geometrically absurd for one hundred years, everyone says that it is an absurd and useless theory, but they can't shake it off. This was my problem, it was a paradigmatic issue, because at the beginning with imaginary numbers the paradox boiled down to a game, daily life with its problems came afterwards, and imaginary numbers were also useful, they could be used to solve problems. But there was no motivation in non-Euclidean geometry; everyone agreed that it was useless. Despite this feeling of utter uselessness always repeated by all mathematicians, non-Euclidean geometry continued to "exist." Then, with no explanation, a non-Euclidean geometry was applied in the theory of relativity. This question is especially important: why did this useless and monstrous theory that was good for nothing hold people's interest? Why couldn't the mathematicians shake it off? In my view, the historical motivation of non-Euclidean geometry lies in the immanent, almost gravitational movement of the spirit towards becoming aware of its freedom; it is indeed thanks to non-Euclidean geometry that the transcendental subject explicitly became aware of its mathematical freedom. Like nature, according to Aristotle, not even the history of mathematics is an episodic tragedy, the work of some miserable author.

8. Relationships with French and Italian Cultures

Polizzi: Even though it is not easy to single out a homeland in your cosmopolitan intellectual wanderings between Hungary, Romania, Germany and France (but also Russia and Italy), France has somehow become your chosen land. We have already recalled your interest in Bourbaki as a typical phenomenon of a postmodern mathematician. But let's get down to the contents, to the *Elements of Mathematics*: for Bourbaki what counted in mathematics was simple and elementary, mathematics expressed the activity of good mathematicians in total freedom, it did not have to be conditioned by logic, but it had to respond to the most abstract axiomatic rigor. You recalled how Bourbaki responded to Russell's statement that mathematics is a part of logic with a postmodern provocation: that Shakespeare and Goethe must also therefore have been parts of grammar.[27] We are in the domain of anti-

[27] I. Toth, "Scienza e scienziati nell'età postmoderna," pp. 323-324; see also Id., "Nicolas Bourbaki, S. A. Vita e opere del 'Matematico policefalo' secondo i dati autentici da lui stesso inventati," *Lettera Matematica Pristem*, 6 (1992).

logicism, so deeply rooted in French mathematics, but with an axiomatic flavor *à la* Hilbert.

Toth: I wrote a text on Bourbaki that was taken from information given to me by Dieudonné and André Weil and by other mathematicians in the group, in which I underline how he put the role of value at the center of mathematical choices. Dieudonné appreciated this surrealist biography and consecrated a *compte-rendu* to it. In the case of Bourbaki the situation is upside-down compared to non-Euclidean geometry; there is a subject that in reality does not exist. This brings up what we were saying earlier and confirms it; while we said that non-Euclidean geometry exists because there is a subject that it knows exists, here we have a subject that doesn't exist as a demographic person but that, despite this, knows of the existence of mathematical theorems, and therefore exists in the mathematical community.

Polizzi: In Italy the seminars you held at the Istituto Italiano per gli Studi Filosofici in Naples, from 1992 (on Kant's *Critique of Pure Reason*) to 2007 (on the paradoxes in Plato's *Parmenides*), aroused great interest.[28] What experience did these "Italian lessons" give you?

Toth: It was a very beautiful and rich experience. They were seminars attended by academics who have a very clear idea of what they are going to study, who enroll and choose what interests them for their training, their studies. They had real reasons for being there, and this made them very stimulating, unlike the seminars at the Collège International de Philosophie which I also held for a few years.

[28] Here are the titles of the seminars held at the Istituto Italiano per gli Studi Filosofici: *La Critica della ragion pura e la ricerca dei fondamenti della geometria nel XVIII secolo* (21-25 September 1992); *La filosofia matematica di Frege e la matematica al tempo di Frege* (12-16 July 1993); *Geometria more ethico. L'alternativa fra geometria euclidea e antieuclidea e la libertà di scelta nel Corpus Aristotelicum* (27 September-1 October 1993); *Libertà e verità: le dimensioni politiche della controversia sulla geometria non-euclidea* (13-17 June 1994); *I numeri del mondo e il mondo dei numeri; il pitagorismo: la matematica nella speculazione filosofica* (10-14 October 1994); *Platone: geometria e filosofia* (10-14 July 1995); *La metagalassia dei numeri e la sua ontologia negativa* (15-19 July 1996); *La Poiesis mathesis. Ontologia della matematica e dell'arte* (4-8 May 1998); *De interpretatione. La geometria non-euclidea nel contesto della "Oratio continua" del commento ad Euclide* (2000); *La bimillenaria controversia sulla geometria non euclidea e la sua ricezione* (13-17 May 2002); *La consapevolezza dell'idea di libertà e i fondamenti della geometria in Aristotele* (12-16 May 2003); *L'addomesticamento dell'infinito: gli argomenti di Zenone e il loro posto nello sviluppo del pensiero filosofico e matematico* (10-14 May 2004); *L'idealismo trascendentale di Kant e il suo ruolo storico e teoretico nella fondazione assiomatica della geometria* (16-20 May 2005); *Diade infinita e l'Uno: essere o non essere degli oggetti aritmetici irrazionali nella filosofia di Platone* (2-6 May 2006); *I paradossi di Zenone nel Parmenide di Platone* (7-11 May 2007).

Polizzi: Again, on your relationship with Italian philosophy, it is well known that some of your important books (*Lo schiavo di Menone* and *Aristotele e i fondamenti assiomatici della geometria*) have been translated thanks to the interest shown by Giovanni Reale, and that you have a close personal and professional relationship with Reale and his research group (among other things in the projects for the Istituto Mediterraneo di Studi Universitari). So, I cannot help but ask you what you think, as an expert on ancient thought with a great knowledge of Plato and Aristotle, of the interpretation of Platonic thought revolving around the "unwritten doctrines" given by Reale himself.

Toth: I have already told you my reflections on the indefinite dyad in the *Parmenides*, which link up perfectly with Reale's reading of the "unwritten doctrines."

(*Translated from the Italian by Karen Whittle*)

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